

STUDENT SUPPORT MATERIAL

Session : 2019-20



तत् त्वं पूषन् अपावृणु
केन्द्रीय विद्यालय संगठन



Class-XII
MATHEMATICS

STUDENT SUPPORT MATERIAL

CLASS XII

MATHEMATICS



Session 2019 – 20

KENDRIYA VIDYALAYA SANGATHAN

NEW DELHI

संतोष कुमार मल्ल, भा.प्र.से.
आयुक्त
Santosh Kumar Mall, I.A.S.
Commissioner



केन्द्रीय विद्यालय संगठन
KENDRIYA VIDYALAYA SANGATHAN
18, संस्थागत क्षेत्र, शहीद जीत सिंह मार्ग, नई दिल्ली-110016
दूरभाष : 91-11-26512579, फ़ैक्स : 91-11-26852680
18, Institutional Area, Shaheed Jeet Singh Marg, New Delhi-110016 (India)
Tel. : 91-11-26512579, Fax : 91-11-26852680
E-mail : commissioner@kvsedu.org, Website : www.kvsangathan.nic.in

A WORD TO MY DEAR STUDENTS

It gives me great pleasure in presenting the Students' Support Material to all KV students of class XII.

The material has been prepared keeping in mind your needs when you are preparing for final exams and wish to revise and practice questions or when you want to test your ability to complete the question paper in the time allotted or when you come across a question while studying that needs an immediate answer but going through the text book will take time or when you want to revise the complete concept or idea in just a minute or try your hand at a question from a previous CBSE Board exam paper or the Competitive exam to check your understanding of the chapter or unit you have just finished. This material will support you in any way you want to use it.

A team of dedicated and experienced teachers with expertise in their subjects has prepared this material after a lot of exercise. Care has been taken to include only those items that are relevant and are in addition to or in support of the text book. This material should not be taken as a substitute to the NCERT text book but it is designed to supplement it.

The Students' Support Material has all the important aspects required by you; a design of the question paper, syllabus, all the units/chapters or concepts in points, mind maps and information in tables for easy reference, sample test items from every chapter and question papers for practice along with previous years Board exam question papers.

I am sure that the Support Material will be used by both students and teachers and I am confident that the material will help you perform well in your exams.

Happy learning!

Santosh Kumar Mall
Commissioner, KVS

FOREWORD

The Students' Support Material is a product of an in-house academic exercise undertaken by our subject teachers under the supervision of subject expert at different levels to provide the students a comprehensive, yet concise, learning support tool for consolidation of your studies. It consists of lessons in capsule form, mind maps, concepts with flow charts, pictorial representation of chapters wherever possible, crossword puzzles, question bank of short and long answer type questions with previous years' CBSE question papers.

The material has been developed keeping in mind latest CBSE curriculum and question paper design. This material provides the students a valuable window on precise information and it covers all essential components that are required for effective revision of the subject.

In order to ensure uniformity in terms of content, design, standard and presentation of the material, it has been fine tuned at KVS Hqrs level.

I hope this material will prove to be a good tool for quick revision and will serve the purpose of enhancing students' confidence level to help them perform better. Planned study blended with hard work, good time management and sincerity will help the students reach the pinnacle of success.

Best of Luck.



U.N. Khaware
Additional Commissioner (Acad.)



STUDENT SUPPORT MATERIAL

ADVISORS

**Shri Santosh Kumar Mall, IAS, Commissioner,
KVS (HQ), New Delhi**

**Sh. Saurabh Jain, IAS
Additional. Commissioner (Admn.)
KVS (HQ), New Delhi.**

**Sh. U.N Khaware,
Additional. Commissioner (Acad)
KVS (HQ), New Delhi.**

CO-ORDINATION TEAM KVS (HQ)

- Dr.E. Prabhakar, Joint Commissioner (Training/Finance)KVS(HQ),New Delhi.
- Smt. InduKaushik, Deputy Commissioner(Acad),KVS (HQ),New Delhi.
- Shri Ravindra Kumar Sharma,Assistant Education Officer, KVS(HQ), New Delhi.

CONTENT TEAM

Mr. S Ajay Kumar, Principal, KV Pattom

Mr. Abraham Mathew, Vice Principal, KV Pattom Shift II

Mr. G Harikumar, PGT (Maths), KV Pattom Shift I

Mrs. Jaya S, PGT (Maths), KV Pattom Shift II

Mrs. Beena Joseph, PGT (Maths), KV SAP

REVIEW TEAM

- Shri C. Karunakaran, Deputy Commissioner, KVS Ernakulam Region.
- Ms Deepti Nair, Assistant Commissioner, KVS Ernakulam Region.
- Shri Ajay Kumar, Principal KV Pattom.
- Shri Abraham Mathew, Vice Principal, KV Pattom Shift-II.
- Shri G. Harikumar, PGT(Maths), KV Shift I
- Smt. Jaya S. PGT (Maths), KV Pattom Shift II
- Smt. Beena Joseph, PGT (Maths), KV SAP

Typing, Type-setting & Designing

M/s Choudhary Printing Press

Near Mohanpur Devi Asthan, Punaichak, Patna-800 023

Mob.: 0943096087, 09835492012 T/F: 0612-2546751

E-mail: choudharyprintingpress@gmail.com

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MATHEMATICS
(Code No. 041)
Session – 2019-20

The Syllabus in the subject of Mathematics has undergone changes from time to time in accordance with growth of the subject and emerging needs of the society. Senior Secondary stage is a launching stage from where the students go either for higher academic education in Mathematics or for professional courses like Engineering, Physical and Biological science, Commerce or Computer Applications. The present revised syllabus has been designed in accordance with National Curriculum Framework 2005 and as per guidelines given in Focus Group on Teaching of Mathematics 2005 which is to meet the emerging needs of all categories of students. Motivating the topics from real life situations and other subject areas, greater emphasis has been laid on application of various concepts.

Objectives

The broad objectives of teaching Mathematics at senior school stage intend to help the students:

- to acquire knowledge and critical understanding, particularly by way of motivation and visualization, of basic concepts, terms, principles, symbols and mastery of underlying processes and skills.
- to feel the flow of reasons while proving a result or solving a problem.
- to apply the knowledge and skills acquired to solve problems and wherever possible, by more than one method.
- to develop positive attitude to think, analyze and articulate logically.
- to develop interest in the subject by participating in related competitions.
- to acquaint students with different aspects of Mathematics used in daily life.
- to develop an interest in students to study Mathematics as a discipline.
- to develop awareness of the need for national integration, protection of environment, observance of small family norms, removal of social barriers, elimination of gender biases.
- to develop reverence and respect towards great Mathematicians for their contributions to the field of Mathematics.

CLASS-XII
(2019-20)

One Paper

Max Marks:80

No.	Units	No. of Periods	Marks
I.	Relations and Functions	30	08
II.	Algebra	50	10
III.	Calculus	80	35
IV.	Vectors and Three - Dimensional Geometry	30	14
V.	Linear Programming	20	05
VI.	Probability	30	08
	Total	240	80
	Internal Assessment		20

Unit-I: Relations and Functions

1. Relations and Functions

15 Periods

Types of relations: reflexive, symmetric, transitive and equivalence relations. One to one and onto functions, composite functions, inverse of a function.

2. Inverse Trigonometric Functions

15 Periods

Definition, range, domain, principal value branch. Graphs of inverse trigonometric functions. Elementary properties of inverse trigonometric functions.

Unit-II: Algebra

1. Matrices

25 Periods

Concept, notation, order, equality, types of matrices, zero and identity matrix, transpose of a matrix, symmetric and skew symmetric matrices. Operation on matrices: Addition and multiplication and multiplication with a scalar. Simple properties of addition, multiplication and scalar multiplication. Non-commutativity of multiplication of matrices and existence of non-zero matrices whose product is the zero matrix (restrict to square matrices of order 2). Concept of elementary row and column operations. Invertible matrices and proof of the uniqueness of inverse, if it exists; (Here all matrices will have real entries).

2. Determinants

25 Periods

Determinant of a square matrix (up to 3 x 3 matrices), properties of determinants, minors, co-factors and applications of determinants in finding the area of a triangle. Adjoint and inverse of a square matrix. Consistency, inconsistency and number of solutions of system of linear equations by examples, solving system of linear equations in two or three variables (having unique solution) using inverse of a matrix.

Unit-III: Calculus

1. Continuity and Differentiability

20 Periods

Continuity and differentiability, derivative of composite functions, chain rule, derivative of inverse trigonometric functions, derivative of implicit functions. Concept of exponential and logarithmic functions.

Derivatives of logarithmic and exponential functions. Logarithmic differentiation, derivative of functions expressed in parametric forms. Second order derivatives. Rolle's and Lagrange's Mean Value Theorems (without proof) and their geometric interpretation.

2. Applications of Derivatives

10 Periods

Applications of derivatives: rate of change of bodies, increasing/decreasing functions, tangents and normals, use of derivatives in approximation, maxima and minima (first derivative test motivated geometrically and second derivative test given as a provable tool). Simple problems (that illustrate basic principles and understanding of the subject as well as real-life situations).

3. Integrals

20 Periods

Integration as inverse process of differentiation. Integration of a variety of functions by substitution, by partial fractions and by parts, Evaluation of simple integrals of the following types and problems based on them.

$$\int \frac{dx}{x^2 \pm a^2}, \int \frac{dx}{\sqrt{x^2 \pm a^2}}, \int \frac{dx}{\sqrt{a^2 - x^2}}, \int \frac{dx}{ax^2 + bx + c}, \int \frac{dx}{\sqrt{ax^2 + bx + c}}$$
$$\int \frac{px + q}{ax^2 + bx + c} dx, \int \frac{px + q}{\sqrt{ax^2 + bx + c}} dx, \int \sqrt{a^2 \pm x^2} dx, \int \sqrt{x^2 - a^2} dx$$
$$\int \sqrt{ax^2 + bx + c} dx, \int (px + q)\sqrt{ax^2 + bx + c} dx$$

Definite integrals as a limit of a sum, Fundamental Theorem of Calculus (without proof). Basic properties of definite integrals and evaluation of definite integrals.

4. Applications of the Integrals

15 Periods

Applications in finding the area under simple curves, especially lines, circles/ parabolas/ellipses (in standard form only), Area between any of the two above said curves (the region should be clearly identifiable).

5. Differential Equations

15 Periods

Definition, order and degree, general and particular solutions of a differential equation.formation of differential equation whose general solution is given.Solution of differential equations by method of separation of variables, solutions of homogeneous differential equations of first order and first degree. Solutions of linear differential equation of the type:

$$\frac{dy}{dx} + py = q, \text{ where } p \text{ and } q \text{ are functions of } x \text{ or constants.}$$
$$\frac{dx}{dy} + px = q, \text{ where } p \text{ and } q \text{ are functions of } y \text{ or constants.}$$

Unit-IV: Vectors and Three-Dimensional Geometry

1. Vectors

15 Periods

Vectors and scalars, magnitude and direction of a vector.Direction cosines and direction ratios of a vector. Types of vectors (equal, unit, zero, parallel and collinear vectors), position vector of a point, negative of a vector, components of a vector, addition of vectors, multiplication of a vector by a scalar, position vector of a point dividing a line segment in a given ratio. Definition, Geometrical Interpretation, properties and application of scalar (dot) product of vectors, vector (cross) product of vectors, scalar triple product of vectors.

2. Three - dimensional Geometry

15 Periods

Direction cosines and direction ratios of a line joining two points.Cartesian equation and vector equation of a line, coplanar and skew lines, shortest distance between two lines.Cartesian and vector equation of a plane.Angle between (i) two lines, (ii) two planes, (iii) a line and a plane.Distance of a point from a plane.

Unit-V: Linear Programming

1. Linear Programming

20 Periods

Introduction, related terminology such as constraints, objective function, optimization, different types of linear programming (L.P.) problems, mathematical formulation of L.P. problems, graphical method of solution for problems in two variables, feasible and infeasible regions (bounded or unbounded), feasible and infeasible solutions, optimal feasible solutions (up to three non-trivial constraints).

Unit-VI: Probability

1. Probability

30 Periods

Conditional probability, multiplication theorem on probability, independent events, total probability, Bayes' theorem, Random variable and its probability distribution, mean and variance of random variable.

Prescribed Books:

- 1) Mathematics Textbook for Class XI, NCERT Publications
- 2) Mathematics Part I - Textbook for Class XII, NCERT Publication
- 3) Mathematics Part II - Textbook for Class XII, NCERT Publication
- 4) Mathematics Exemplar Problem for Class XI, Published by NCERT
- 5) Mathematics Exemplar Problem for Class XII, Published by NCERT
- 6) Mathematics Lab Manual class XI, published by NCERT
- 7) Mathematics Lab Manual class XII, published by NCERT

<http://www.ncert.nic.in/exemplar/labmanuals.html>

MATHEMATICS (Code No. - 041)
QUESTION PAPER DESIGN CLASS - XII
(2019 - 20)

Time: 3 hours

Max. Marks: 80

S. No.	Typology of Questions	Very Short Answer-Objective type (VSA) (1 Mark)	Short Answer-I (SA) (2 Marks)	Long Answer-I (SA) (4 Marks)	Long Answer (LA) (6 Marks)	Total Marks	% Weightage
1	Remembering: Exhibit memory of previously learned material by recalling facts, terms, basic concepts, and answers.	4	1	1	1	16	20
2	Understanding: Demonstrate understanding of facts and ideas by organizing, comparing, translating, interpreting, giving descriptions, and stating main ideas	6	2	3	1	28	35
3	Applying: Solve problems to new situations by applying acquired knowledge, facts, techniques and rules in a different way.	6	2	1	1	20	25
4	Analysing : Examine and break information into parts by identifying motives or causes. Make inferences and find evidence to support generalizations Evaluating: Present and defend opinions by making judgments about information, validity of ideas, or quality of work based on a set of criteria. Creating: Compile information together in a different way by combining elements in a new pattern or proposing alternative solutions	4	1	1	1	16	20
Total		20x1 =20	6x2 =12	6x4=24	4x6=24	80	100

QUESTION-WISE BREAK-UP

Type of Question	Mark per Question	Total No. of Questions	Total Marks
VSA	1	20	20
SA	2	6	12
LA I	4	6	24
LA II	6	4	24
Total		36	80

1. *No chapter wise weightage. Care to be taken to cover all the chapters*
2. *Suitable internal variations may be made for generating various templates keeping the overall weightage to different form of questions and typology of questions same.*

Choice(s):

There will be no overall choice in the question paper.

However, 33% internal choices will be given.

INTERNAL ASSESSMENT	20 MARKS
Periodic Tests (Best 2 out of 3 tests conducted)	10 Marks
Mathematics Activities	10 Marks

Note: For activities NCERT Lab Manual may be referred

Conduct of Periodic Tests:

Periodic Test is a Pen and Paper assessment which is to be conducted by the subject teacher. The format of periodic test must have questions items with a balance mix, such as, very short answer (VSA), short answer (SA) and long answer (LA) to effectively assess the knowledge, understanding, application, skills, analysis, evaluation and synthesis. The subject teacher will have the liberty of incorporating any other types of questions too. The modalities of the PT are as follows:

- a) **Mode:** The periodic test is to be taken in the form of pen-papertest.
- b) **Schedule:** In the entire Academic Year, three Periodic Tests in each subject may be conducted as follows:

Test	Pre Mid-term (PT-I)	Mid-Term (PT-II)	Post Mid-Term (PT-III)
Tentative Month	July-August	November	December-January

This is only a suggestive schedule and schools may conduct periodic tests as per their convenience. The winter bound schools would develop their own schedule with similar time gaps between two consecutive tests.

- c) **Average of Marks:** Once schools complete the conduct of all the three periodic tests, they will convert the weightage of each of the three tests into ten marks each for identifying best two tests. The best two will be taken into consideration and the average of the two shall be taken as the final marks for PT.
- d) The school will ensure simple documentation to keep a record of performance
- e) **Sharing of Feedback/Performance:** The students' achievement in each test must be shared with the students and their parents to give them an overview of the level of learning that has taken place during different periods. Feedback will help parents formulate interventions (conducive ambience, support materials, motivation and morale-boosting) to further enhance learning. A teacher, while sharing the feedback with student or parent, should be empathetic, non- judgmental and motivating. It is recommended that the teacher share best examples/performances with the class to motivate all learners.

Assessment of Activity Work:

Throughout the year any 10 activities shall be performed by the student from the activities given in the NCERT Laboratory Manual for the respective class (XI or XII) which is available on the link : <http://www.ncert.nic.in/exemplar/labmanuals.html> A record of the same may be kept by the student. An year end test on the activity may be conducted at the School Level.

The weightage are as under:

- The activities performed by the student through out the year and record keeping : 5 marks
- Assessment of the activity performed during the year end test: 3 marks
- Viva-voce : 2 marks

SAMPLE QUESTION PAPER
MATHEMATICS
CLASS XII (2019-20)

Time allowed: 3 hours

Maximum Marks: 80

General Instructions:

- (i) All questions are compulsory.
- (ii) This question paper contains 36 questions.
- (iii) Question 1- 20 in Section A carrying 1 mark each.
- (iv) Questions 21-26 in Section B carrying 2 marks each.
- (v) Questions 27-32 in Section C carrying 4 marks each.
- (vi) Questions 33-36 in Section D carrying 6 marks each.

SECTION A

1. Find the gof if $f(x) = 8x^3$ and $g(x) = x^{1/3}$

2. Find the value of $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$

3. Find the principal value of $\sec^{-1}(2/\sqrt{3})$

4. Find the cofactor of element a_{12} in the following , $\begin{bmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & 7 \end{bmatrix}$

5. Examine the continuity of the function $f(x) = 2x^2 - 1$ at $x = 3$.

6. Find the rate of change of the area of the circle with respect to its radius when $r = 4\text{cm}$.

(A) 4 (B) 8π (C) π^2 (D) 2π

7. Differentiate the function with respect to x , $\sec(\tan\sqrt{x})$

8. Integrate $\int \frac{dx}{x \log x}$

(A) $\log x + C$ (B) $\log(\log x) + C$ (C) $\log(\log(\log x)) + C$ (D) none

9. Differentiate it with respect to x , $\log(\log x)$, where $x > 1$

10. If $A = \begin{bmatrix} a & 2 \\ 2 & a \end{bmatrix}$ and $|A|^3 = 125$ then find "a"

(A) ± 3 (B) 5 (C) 2 (D) ± 2

11. Find the slope of the tangent on the curve $y = 3x^4 - 4x$ at $x = 4$.

12. If $P(A) = 1/2$, $P(B) = 0$ then find $P(A/B)$

13. Find $\frac{dy}{dx}$ if x and y are connected parameters $x = 4t$, $y = 4/t$

14. If $f: \mathbb{R} \rightarrow \mathbb{R}$ be given by $f(x) = (3 - x^3)^{1/3}$ then find f of

15. For what value of 'a' the vectors $2\hat{i} - 3\hat{j} + 4\hat{k}$ and $a\hat{i} + 6\hat{j} - 8\hat{k}$ are collinear

16. Using differentials, find the approximate value of $\sqrt{25.3}$ up to 2 places of decimal.

17. Evaluate $\int_2^3 \frac{1}{x} dx$

18. Given that $P(\bar{A}) = 0.4$, $P(B) = 0.2$ and $P(A/B) = 0.5$ find $P(A \cup B)$.

19. Two tailors, A and B, earn Rs. 300/- and Rs 400/- per day respectively. A can stitch 6 shirts and 4 pairs of trousers while B can stitch 10 shirts and 4 pairs of trousers per day. To find how many days should each of them work and if it is decided to produce at least 60 shirts and 32 pairs of trousers at a minimum labour cost, formulate this as an LPP.

20. If θ is the angle between any two vectors \vec{a} and \vec{b} then $|\vec{a} \cdot \vec{b}| = |\vec{a} \times \vec{b}|$ then θ is equal to

(A) 0 (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) π

SECTION B

21. If $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 7 \end{bmatrix}$, then find the value of $A^2 - 3A + 2I$.

22. If the rate of change of volume of a sphere is equal to the rate of change of its radius then find its radius.

23. Differentiate $\tan^{-1} \left[\frac{\sqrt{1+x^2}-1}{x} \right]$ with respect to x .

24. Evaluate $\int x\sqrt{x+2} dx$

25. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 4\hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$, find a vector of magnitude 6 units which is parallel to the vector $2\vec{a} - \vec{b} + 3\vec{c}$

26. Find $\int e^{2x} \left(\frac{1 - \sin 2x}{1 - \cos 2x} \right) dx$

SECTION C

27. For what values of a and b , the function f defined as:

$$f(x) = \begin{cases} 3ax + b, & \text{if } x < 1 \\ 11, & \text{if } x = 1 \\ 5ax - 2b, & \text{if } x > 1 \end{cases} \quad \text{is continuous at } x=1.$$

OR

Find the values of k so that the function f is continuous at $x = \pi/2$

$$f(x) = \begin{cases} \frac{k \cos x}{\pi - 2x}, & \text{if } x \neq \frac{\pi}{2} \\ 3, & \text{if } x = \frac{\pi}{2} \end{cases}$$

28. Find the image of the point $(1, 2, 3)$ in the plane $x + 2y + 4z = 38$.

29. Consider $f : R_+ \rightarrow [4, \infty)$ given by $f(x) = x^2 + 4$. Show that f is invertible with the inverse f^{-1} of given by $f^{-1}(y) = \sqrt{y - 4}$, where R_+ is the set of all non-negative real number.

OR

Show that the relation R in the set $A = \{1, 2, 3, 4, 5\}$ given by $R = \{(a, b) : |a - b| \text{ is even}\}$, is an equivalence relation. Show that all the element of $\{1, 3, 5\}$ are related to each other and all the element of $\{2, 4\}$ are related to each other. But no element of $\{1, 3, 5\}$ is related to any element of $\{2, 4\}$.

30. An aeroplane can carry a maximum of 200 passengers. A profit of Rs. 1000 is made on each executive class ticket and a profit of Rs. 600 is made on each economy class ticket. The airline reserves at least 20 seats for executive class. However at least 4 times as many passengers prefer to travel by economy class than by the executive class. Determine how many tickets of each type must be sold in order to maximize the profit for the airline. What is the maximum profit?

31. Evaluate $\int \frac{x^3}{x^4 + 3x^2 + 2}$

OR

Evaluate $\int_1^3 (3x^2 + 2x) dx$ as the limit of a sum

32. The side of an equilateral triangle is increasing at the rate of 2cm/sec at what rate is its area increasing when the side of triangle is 20 cm.

SECTION D

33. Find the length and foot of the perpendicular drawn from the point (2,-1,5) on the line

$$\frac{x-11}{10} = \frac{y+2}{-4} = \frac{z+8}{-11}$$

34. Find the area of the region bounded by

$$\{(x, y) | y^2 \leq 6ax \text{ and } x^2 + y^2 \leq 16a^2\}, \text{ using method of integration}$$

35. From a lot of 15 bulbs which include 5 defectives, a sample of 4 bulbs is drawn one by one with replacement. Find the probability distribution of number of defective bulbs. Hence find the mean of the distribution.

36. Solve the matrix equation, $A \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 0 \\ 2 & 3 \end{bmatrix}$ using concept of inverse.

OR

Using properties of determinants prove that

$$\begin{bmatrix} a + bx & c + dx & p + qx \\ ax + b & cx + d & px + q \\ u & v & w \end{bmatrix} = (1-x^2) \begin{bmatrix} a & c & p \\ b & d & q \\ u & v & w \end{bmatrix}$$

BLUE PRINT
MATHEMATICS
CLASS XII (2019-20)

Time allowed: 3 hours

Maximum Marks: 80

	VSA (20)		SA (12)		LA1 (24)		LA2 (24)		TOTAL (80)
	I mark		2 mark		4 mark		6mark		
1.Relations and Functions	2	2		0	1	4		0	6
2.InverseTrigonometric Functions	2	2		0		0		0	2
3. Matrices	2	2	1	2		0		0	4
4. Determinants		0		0		0	1	6	6
5.Continuity and Differentiability	4	4	1	2	1	4		0	10
6.Applications of Derivatives	3	3	1	2	1	4		0	9
7.Integrals	2	2	2	4	1	4		0	10
8.Applications of the Integrals		0		0		0	1	6	6
9.Vectors	2	2	1	2		0		0	4
10.Three Dimensional Geometry		0		0	1	4	1	6	10
11.Linear programming	1	1		0	1	4		0	5
12.Probability	2	2	1	2	1	4		0	8
	20	20	6	12	6	24	4	24	80

**MARKING SCHEME
MATHEMATICS
CLASS XII (2019-20)**

SECTION A		
1	$\text{gof} = 2x$	1
2	$\pi/3$	1
3	$\pi/6$	1
4	$A_{12} = -38$	1
5	Continuous	1
6	8π	1
7	$\sec(\tan\sqrt{x}) \cdot \tan(\tan\sqrt{x}) \cdot (\sec^2\sqrt{x})^{\frac{1}{2\sqrt{x}}}$	1
8.	B	1
9.	$\frac{1}{x \log x}$	1
10	$a = \pm 3$	1
11	$(\frac{dy}{dx})_{x=4} = 764$	1
12	P(A/B) is not defined.	1
13	$\frac{dy}{dx} = -1/t^2$	1
14	x	1
15	$a = -4$	1
16	5.03	1
17	$\log(3/2)$	1
18	0.7	1
19	$Z = 300x + 400y$ $6x + 10y \geq 60$ $4x + 4y \geq 32, x, y \geq 0$ where X and Y denotes the number of days and Z denotes the labour cost.	1
20	$\frac{\pi}{4}$	1
SECTION B		
21	If $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 7 \end{bmatrix}$, then find the value of $A^2 - 3A + 2I$.	
	$A^2 = \begin{bmatrix} 5 & -1 & 9 \\ 9 & -2 & 26 \\ 7 & -8 & 47 \end{bmatrix}, 3A = \begin{bmatrix} 6 & 0 & 3 \\ 6 & 3 & 9 \\ 3 & -3 & 21 \end{bmatrix}, 2I = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$	$1/2 + 1/2 + 1/2$
	$A^2 - 3A + 2I = \begin{bmatrix} 1 & -1 & 6 \\ 3 & -3 & 17 \\ 4 & -5 & 28 \end{bmatrix}$	$1/2$

22	$\frac{dv}{dt} = \frac{dr}{dt}$ $4\pi r^2 \frac{dr}{dt} = \frac{dr}{dt}$ $r = 1/2\sqrt{\pi}$	$\frac{1}{2}$ $\frac{1}{2}$ 1
23	Put $x = \tan t$ $\tan^{-1} \left[\frac{\sqrt{1 + \tan^2 t} - 1}{\tan t} \right]$ $\tan^{-1} \left[\frac{\sec t - 1}{\tan t} \right] = \tan^{-1} \tan \frac{t}{2} = t/2$ $= (\tan^{-1} x)/2$ $\frac{dy}{dx} = \frac{1}{2(1+x^2)}$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
24	$I = \int x\sqrt{x+2} dx$ $I = \int ((x+2) - 2)\sqrt{x+2} dx$ $I = \int (x+2)\sqrt{x+2} dx - \int 2\sqrt{x+2} dx$ $I = \frac{2}{5}(x+2)^{\frac{5}{2}} - \frac{4}{3}(x+2)^{\frac{3}{2}} + C$	$\frac{1}{2}$ $\frac{1}{2}$ 1
25	$2\vec{a} - \vec{b} + 3\vec{c} = \hat{i} - 2\hat{j} + 2\hat{k}$ <p>Vector of magnitude $6 = 2\hat{i} - 4\hat{j} + 4\hat{k}$</p>	1 1
26	Let $2x = t$ $\frac{1}{2} \int e^t (\frac{1}{2} \operatorname{cosec}^2 t/2 - \cot t/2) dt$ $= -\frac{1}{2} e^{2x} \cot x + c$	$\frac{1}{2}$ 1 $\frac{1}{2}$
SECTION C		
27	On finding LHL = $3a + b$ On getting RHL = $5a - 2b$ As $f(x)$ is continuous at $x = 1$ LHL = RHL = $f(1)$ $3a + b = 5a - 2b = 11$ On getting $a = 3$ and $b = 2$.	1 1 1 1
OR		
	For $f(x)$ to be continuous at $x = 0$, LHL = RHL = $f(x)$	1

	<p>Now $f\left(\frac{\pi}{2}\right) = 3 \dots\dots(i)$</p> <p>LHL = $\lim_{x \rightarrow \frac{\pi}{2}^-} f(x) = \lim_{x \rightarrow \frac{\pi}{2}^-} \frac{k \cos x}{\pi - 2x} = \lim_{h \rightarrow 0} \frac{k \sin 2h}{2h} = \frac{k}{2} \dots\dots(ii)$ (</p> <p>We assume $x = \pi/2 - h$ so that $x \rightarrow \pi/2, h \rightarrow 0$)</p> <p>RHL = $\lim_{x \rightarrow \frac{\pi}{2}^+} f(x) = \lim_{x \rightarrow \frac{\pi}{2}^+} \frac{k \cos x}{\pi - 2x} = \lim_{h \rightarrow 0} \frac{-k \sin 2h}{-2h} = \frac{k}{2} \dots\dots(iii)$</p> <p>$k/2 = 3$</p> <p>$k = 6$</p>	1 1 1 1
28	<p>Let image of point P(1, 2, 3) be Q(α, β, γ)</p> <p>Now, the equation of line passing through the point P and perpendicular to given plane is</p> $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{4} = \lambda$ <p>$x = \lambda + 1, y = 2\lambda + 2, z = 4\lambda + 3$</p> <p>Let M be point of intersection of line and plane. Then, coordinates of M are ($\lambda + 1, 2\lambda + 2, 4\lambda + 3$) Since points lie on plane $\lambda + 1, +2(2\lambda + 2) + 4(4\lambda + 3) = 38$ $\lambda = 1$ Thus, coordinates of M are (2, 4, 7)</p> <p>Also coordinates of M are $\left(\frac{\alpha+1}{2}, \frac{\beta+2}{2}, \frac{\gamma+3}{2}\right)$, Hence $\left(\frac{\alpha+1}{2}, \frac{\beta+2}{2}, \frac{\gamma+3}{2}\right) = (2, 4, 7)$ Which gives $\alpha = 3, \beta = 6$ and $\gamma = 11$ Hence image of point P is (3,6,11)</p>	1 1 1 1 1
29	<p>To show : R is Reflexive, Symmetric & Transitive</p> <p>For proving all the elements of {1,3,5} are related to each other</p> <p>For proving all the elements of {2,4} are related to each other</p> <p>For proving no element of {1,3,5} is related to any element of {2,4}.</p> <p style="text-align: center;">OR</p> <p>To show : f is one - one</p> <p>To show : f is onto</p> <p>To show : $f^{-1}(y) = \sqrt{y-4}$</p>	1 1 1 1 1 2 1
30	<p>Let the airline sell x tickets of executive class and y tickets of economy class.</p> <p>The mathematical formulation of the given problem is as follows.</p> <p>Maximize $z = 1000x + 600y \dots (1)$</p>	1 1

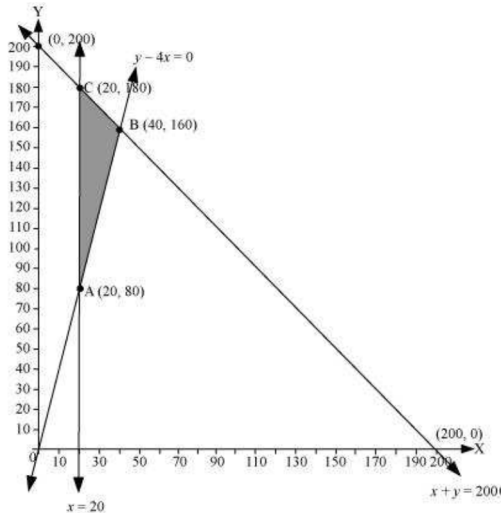
$$x + y \leq 200 \quad \dots(2)$$

$$x \geq 20 \quad \dots(3)$$

$$y - 4x \geq 0 \quad \dots(4)$$

$$x, y \geq 0 \quad \dots(5)$$

Subject to the constraints, the feasible region determined by the constraints is as follows.



The corner points of the feasible region are A (20, 80), B (40, 160), and C (20, 180).

The values of z at these corner points are as follows.

Corner point	$z = 1000x + 600y$	
A (20, 80)	68000	
B (40, 160)	136000	→ Maximum
C (20, 180)	128000	

The maximum value of z is 136000 at (40, 160).

Thus, 40 tickets of executive class and 160 tickets of economy class should be sold to maximize the profit and the maximum profit is Rs 136000.

1

1

31 Let $x^2 = t$

$$\int \frac{x^3}{x^4 + 3x^2 + 2} dx = \int \frac{t}{2(t^2 + 3t + 2)} dt$$

$$= \frac{1}{2} \int \frac{t}{(t+1)(t+2)} dt$$

$$= \frac{1}{2} [2 \log(x^2 + 2) - \log(x^2 + 1)]$$

OR

Formula

$$a=1, b=3, nh=2$$

Finding $f(1), f(1+h), f(1+2h)$

Final answer = 34

2

2

2

2

	$= \begin{bmatrix} -7/2 & 3/2 \\ 0 & 0 \\ 1/2 & 1/2 \end{bmatrix}$	1
	OR	
	Performing $R_1 \rightarrow R_1 - xR_2$ and taking $(1-x^2)$ outside	2
	$\Delta = (1-x^2) \begin{vmatrix} a & c & p \\ ax + b & cx + d & px + q \\ u & v & w \end{vmatrix}$	2
	Apply $R_2 \rightarrow R_2 - xR_1$ and getting the result	2

List of Important Formulas

RELATIONS & FUNCTIONS

- (a) A relation in set A is a subset of $A \times A$. We also write it as $R = \{(a, b) \in A \times A ; a R b\}$.

(b) For relation R in set A , R^{-1} is inverse relation if $a R^{-1} b \Rightarrow b R a$.
- A relation R in a set A is said to be reflexive, if $(a, a) \in R$, for every $a \in A$ or we say $a R a$, for every $a \in A$.
- A relation R in a set A is said to be symmetric, if $(a, b) \in R \Rightarrow (b, a) \in R$, for all $a, b \in A$. We can also say $a R b, b R a$ for every $a, b \in A$.
- A relation R in a set A is said to be transitive, if $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$ for every $a, b, c \in A$. We can also say $a R b, b R c \Rightarrow a R c$, for all $a, b, c \in A$.
- A relation in a set A is said to be an equivalence relation if relation R is reflexive, symmetric and transitive.
- A function f is a rule from set A to set B which assigns to each element of set A , a unique element of set B . Set A is called the domain of the function f set B is known as its co-domain. The set of values from set B which are actually taken by the function f is called the range of the function f .
We denote it as $f: A \rightarrow B$, if $x \in A$ then $f(x) \in B$.
- A function whose domain and co-domain are the sets of real numbers is known as a real valued function, i.e. $f: R \rightarrow R$.
- One-one function: a function $f: A \rightarrow B$ is said to be one-one (or injective), if the images of distinct elements of A under the rule f are distinct in B , i.e. for every $a, b \in A, a \neq b \Rightarrow f(a) \neq f(b)$ or we also say that $f(a) = f(b) \Rightarrow a = b$.
- Onto function $f: A \rightarrow B$ is said to be onto (or surjective), if every element of B is image of some element of A under the rule f , i.e. for every $b \in B$, there exists an element $a \in A$ such that $f(a) = b$.
NOTE: -A function is onto if and only if Range of function $f = B$.
- Bijjective function: A function $f: A \rightarrow B$ is said to be bijective if it is both one-one and onto.
- Composition of function: Let $f: A \rightarrow B$ and $g: B \rightarrow C$ be two given functions. Then the composition of function from A to C is denoted by $g \circ f$. We define $g \circ f$ as $g \circ f(x) = g\{f(x)\} \forall x \in A$. For working on element x first we apply f rule and whatever result is obtained in set B . we apply g rule on it to get the required result in set C .
- A function $f: A \rightarrow B$ is said to be invertible, If there exists a function $g: B \rightarrow A$ such that $g \circ f = f \circ g = I$. The function g is called the inverse of function f and is denoted by f^{-1} .



RELATIONS AND FUNCTIONS
MULTIPLE CHOICE QUESTIONS

1. Let T be the set of all triangles in the Euclidean plane, and let a relation R on T be defined as aRb if a is congruent to b , $a, b \in T$. Then R is
 (A) reflexive but not transitive (B) transitive but not symmetric
 (C) equivalence (D) none of these
2. If a relation R on the set $\{1,2,3\}$ be defined by $R = \{(1,2)\}$ then R is
 (A) reflexive (B) transitive
 (C) symmetric (D) none of these
3. If $f: R \rightarrow R$ given by $f(x) = (3 - x^3)^{\frac{1}{3}}$, find $f \circ f(x)$
 (A) $x^{1/3}$ (B) x^3 (C) x (D) $(3 - x^3)$
4. Let $A = \{1,2,3\}$. The number of equivalence relations containing $(1,2)$ is
 (A) 1 (B) 2 (C) 3 (D) 4
5. Let $f: R \rightarrow R$ defined by $f(x) = x^4$. Choose the correct answer
 (A) *f is one one onto* (B) *f is many one onto* (C) *f is oneone but not onto*
 (D) *f is neither one-one nor onto*
6. Let $f: R \rightarrow R$ defined by $f(x) = 3x$. Choose the correct answer
 (A) *f is one one onto* (B) *f is many one onto* (C) *f is one-one but not onto*
 (D) *f is neither one-one nor onto*
7. If $A = \{1,2,3\}$, $B = \{4,6,9\}$ and R is a relation from A to B defined by ' x is smaller than y '. The range of R is
 (A) $\{1, 4, 6, 9\}$ (B) $\{4,6,9\}$ (C) $\{1\}$ (D) none of these
8. The relation $R = \{(1,1), (2,2), (3,3)\}$ on $\{1,2,3\}$ is
 (A) *symmetric only* (B) *reflexive only* (C) *transitive only* (D) *an equivalence relation*
9. Let $A = \{1,2,3\}$ and consider the relation $R = \{(1,1), (2,2), (3,3), (1,2), (2,3), (1,3)\}$ then R is
 (A) reflexive but not symmetric (B) reflexive but not transitive
 (C) symmetric and transitive (D) neither symmetric nor transitive
10. Let us define a relation R in R as $a R b$ if $a \geq b$. Then R is
 (A) an equivalence relation (B) reflexive, transitive but not symmetric
 (C) symmetric, transitive but not reflexive (D) neither transitive nor reflexive but symmetric

ANSWERS

- | | |
|------|-------|
| 1. C | 6. A |
| 2. B | 7. B |
| 3. C | 8. D |
| 4. B | 9. A |
| 5. D | 10. B |

INVERSE TRIGONOMETRIC FUNCTIONS

FUNCTION	DOMAIN	RANGE (P,VALUE)
$\text{Sin}^{-1}x$	$[-1,1]$	$[-\pi/2,\pi/2]$
$\text{Cos}^{-1}x$	$[-1,1]$	$[0,\pi]$
$\text{tan}^{-1}x$	\mathbf{R}	$(-\pi/2,\pi/2)$

- 1) (i) $\text{Sin}^{-1}(\sin x)=x, [-\pi/2,\pi/2]$
 (ii) $\text{Cos}^{-1}(\cos x)=x, [0,\pi]$
 (iii) $\text{tan}^{-1}(\tan x)=x, (-\pi/2,\pi/2)$

- 2) (i) $\text{Sin}^{-1}(-x)=-\text{Sin}^{-1}(x), x \in [-1,1]$
 (ii) $\text{cosec}^{-1}(-x)=-\text{cosec}^{-1}x, x \geq 1$
 (iii) $\text{tan}^{-1}(-x)=-\text{tan}^{-1}(x), x \in \mathbf{R}$
 (iv) $\text{Cos}^{-1}(-x)=\pi-\text{cos}^{-1}(x), x \in [-1,1]$
 (v) $\text{sec}^{-1}(-x)=\pi-\text{sec}^{-1}(x), |x| \geq 1$
 (vi) $\text{cot}^{-1}(-x)=\pi-\text{cot}^{-1}x, x \in \mathbf{R}$

- 3) (i) $\text{cosec}^{-1}(x)=\text{sin}^{-1}(\frac{1}{x}), x \geq 1, x \leq -1$
 (ii) $\text{sec}^{-1}(-x)=\text{cos}^{-1}(\frac{1}{x}), x \geq 1, x \leq -1$
 (iii) $\text{cot}^{-1}(-x)=\text{tan}^{-1}(\frac{1}{x}), x > 0$

- 4) (i) $\text{Sin}^{-1}x + \text{Cos}^{-1}x = \frac{\pi}{2}, x \in [-1,1]$
 (ii) $\text{cosec}^{-1}(-x) + \text{sec}^{-1}(-x) = \frac{\pi}{2}, |x| \geq 1$
 (iii) $\text{tan}^{-1}(x) + \text{cot}^{-1}(x) = \frac{\pi}{2}, x \in \mathbf{R}$

- 5) $\text{tan}^{-1}x + \text{tan}^{-1}y = \text{tan}^{-1}\frac{x+y}{1-xy}, \text{ if } xy < 1$
 $\text{tan}^{-1}x + \text{tan}^{-1}y = \pi + \text{tan}^{-1}\frac{x+y}{1-xy}, \text{ if } x, y > 0 \text{ and } xy > 1$

- 6) $\text{tan}^{-1}x + \text{tan}^{-1}y = \text{tan}^{-1}\frac{x-y}{1+xy}, xy > -1$
- 7) $2 \text{tan}^{-1}x = \text{sin}^{-1}\frac{2x}{1+x^2}, -1 < x < 1$
 $2 \text{tan}^{-1}x = \text{cos}^{-1}\frac{1-x^2}{1+x^2}, x \geq 0$
 $2 \text{tan}^{-1}x = \text{tan}^{-1}\frac{2x}{1+x^2}, -1 < x < 1$

ANSWER

Domain,co-domain,range of a relation. Types of relation.

Level:-I

1. $R = \{(3,5), (4,4), (5,3)\}$

$D = \{3,4,5\}$

$Co - domain = A$

$Range = \{3,4,5\}$

2. Transitive but not reflexive

Level:- II

3. Transitive but not reflexive

Level:- III

1. $\{2,6,10\}$

6. T_1 is related to T_3

One-one, onto & inverse of a function

Level:-I

4. $f\left(\frac{1}{x}\right) = \frac{1}{x^2} + x^2$

5. $f^{-1}(x) = \frac{1+x}{1-x}, x \neq 1$

Level:- II

2. $\frac{2y-5}{3}$ 3. YES 6. $\vec{f} = \frac{4y+3}{6y-4}$ 7. 6

Level:- III

4. $\frac{3y-2}{y-1}, y \neq 1$

Composition of function

Level:-I

1. $x^2 - 6x^3 + 10x^2 - 3x$ 2. $|5x - 2|$ 3. 7

Level:-II

1. $\frac{y-7}{10}$ 2. x 3. 2,1 4. x 5. $(2x - 3)^2$



ANSWERS

Conditional Probability

Level-I

1. $\frac{4}{11}$, $\frac{4}{5}$ and $\frac{2}{3}$

2. (a) $P(E/F) = 1/2$ (b) $P(G/E) = 2/3$ (c) $P(E \cup F/G) = \frac{3}{4}$

Level-II

1. $\frac{1}{2}$ 2. 1

Level-III

1. $\frac{1}{3}$ 2. $\frac{2}{5}$

Multiplication theorem on probability and independent events

Level-I

1. $\frac{1}{2}$ and $\frac{1}{4}$

Level-II

1. $\frac{5}{65}$ 2. (a) $\frac{13}{21}$ (b) $\frac{10}{21}$

Level-III

1. $\frac{3}{5}$ 2. $\frac{27}{50}$

Bayes' theorem

Level-I

1. $\frac{\binom{25}{5}}{\binom{52}{5}}$ 2. (0.22) 3. (1/4) 4. (42%)

Level-II

1. (3/8) 2. (8/11) 3. $\frac{24}{29}$

Level-III

1. $\frac{1}{11}$, $\frac{7}{11}$ 2. $\frac{3}{5}$ 3. $\frac{99}{1080}$ 4. $\frac{19}{42}$

Random Variables & Probability distribution, means, variance and standard deviation.

Level-I

1. $P(x = 0) = 144/169$, $P(X = 1) = 24/169$, $P(x = 2) = 1/169$
 2. Mean = $\frac{1}{2}$ Variance = $\frac{1}{4}$
 3. Mean = $\frac{9}{10}$ Variance = $\frac{49}{100}$

Level-II

1. Mean = $\frac{2}{30}$ Variance : $\frac{400}{2873}$
 2. (i) $\frac{1}{10}$ (ii) $\frac{19}{100}$ (iii) $\frac{4}{5}$

Level-III

1.

x	0	1	2		
$P(x)$	$\frac{9}{16}$	$\frac{3}{8}$	$\frac{1}{16}$		

2.

x	2	3	4	5	6
$P(x)$	$\frac{2}{30}$	$\frac{4}{30}$	$\frac{6}{30}$	$\frac{8}{30}$	$\frac{10}{30}$



MULTIPLE CHOICE QUESTIONS

1. If $\sin \{ \sin^{-1}(\frac{1}{2}) + \cos^{-1}x \} = 1$, then the value of x is
 (A) $\frac{\pi}{2}$ (B) 1 (C) 0 (D) $\frac{1}{2}$
2. If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi/2$, $x, y, z > 0$, then the value of $xy+yz+zx$ is
 (A) $\frac{\pi}{2}$ (B) 1 (C) 0 (D) not defined
3. If $\sin^{-1}(1-x) - 2\sin^{-1}(x) = \frac{\pi}{2}$, then x is equal to
 (A) $0, \frac{1}{2}$ (B) $1, \frac{1}{2}$ (C) 0 (D) $\frac{1}{2}$
4. $\cos^{-1}(\cos(\frac{7\pi}{6}))$ is equal to
 (A) $\frac{7\pi}{6}$ (B) $\frac{5\pi}{6}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{3}$
5. If $3\sin^{-1}(\frac{2x}{1+x^2}) - 4\cos^{-1}(\frac{1-x^2}{1+x^2}) + 2\tan^{-1}\frac{2x}{1-x^2} = \frac{\pi}{3}$ then x is equal to
 (A) $\frac{1}{\sqrt{3}}$ (B) $\sqrt{3}$ (C) 1 (D) not defined
6. Which of the following is the principal value branch of $\cos^{-1}x$
 (A) $[-\frac{\pi}{2}, \frac{\pi}{2}]$ (B) $(0, \pi)$ (C) $[0, \pi]$ (D) $(0, \pi) - \{ \frac{\pi}{2} \}$
7. If $3\tan^{-1}x + \cot^{-1}x = \pi$, then x equals
 (A) 0 (B) -1 (C) 1 (D) $\frac{1}{2}$
8. The value of the expression $2\sec^{-1}2 + \sin^{-1}(\frac{1}{2})$ is
 (A) $\frac{\pi}{6}$ (B) $\frac{5\pi}{6}$ (C) 1 (D) $\frac{7\pi}{6}$
9. $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}$ is true for
 (A) $x, y \in R$ (B) $|x| > 1, |y| > 1$ (C) $|x| < 1, |y| < 1$ (D) $xy < 1$
10. $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$ is equal to
 (A) π (B) $\frac{\pi}{3}$ (C) $-\frac{\pi}{3}$ (D) $\frac{2\pi}{3}$

ANSWERS

- 1.D 2.B 3.C 4.B 5.A 6.C 7.C 8.B 9.D 10.C

MATRICES AND DETERMINANTS

MULTIPLE CHOICE QUESTIONS

1. If A is a square matrix of order 3 with $|A| = 4$ then find $|2A|$
 (a) 12 (b) 32 (c) 16 (d) none of these
2. If A is a square matrix of order 3 with $|\text{adj}A| = 25$ then find $|A|$
 (a) 25 (b) ± 125 (c) ± 5 (d) 15
3. If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, and $A + A' = I$ then α is
 (a) π (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$
4. If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$ then value of x is
 (a) 3 (b) ± 6 (c) 8 (d) -2
5. The total number of possible matrices of order 3×3 with each entry 2 or 1 is
 (a) 27 (b) 18 (c) 81 (d) 512
6. If A is a square matrix such that $A^2 = I$ then $(A - I)^3 + (A + I)^3 - 7A$ is equal to
 (a) A (b) I (c) 3A (d) I-A
7. If area of a triangle is 35 square units with vertices (2,-6), (5,4) and (k,4), then k is
 (a) 12 (b) -2 (c) -12, -2 (d) 12, -2
8. If A is a square matrix of order 3, and $|A| = 4$ then $|\text{adj}A|$
 (a) 4 (b) 12 (c) 16 (d) 64
9. Let A be a 5×7 matrix, then each column of A contains
 (a) 7 elements (b) 5 elements (c) 35 elements (d) none of these
10. If A is any square matrix of order n, then $A(\text{adj}A)$ is equal to
 (a) $|A|I$ (b) I (c) 0 (d) none of these

ANSWERS

- 1.b 2.c 3.b 4.b 5.d 6.a 7.d (8)c (9)b (10)a

CONTINUITY AND DIFFERENTIABILITY
MULTIPLE CHOICE QUESTIONS

- Let $f(x) = |x| + |x - 1|$
then $f(x)$ is continuous at
(a) $x=0$, and $x=1$ (b) $x=0$, but not at $x=1$ (c) $x=1$ but not at $x=0$ (d) none of these
- If $f(x) = \begin{cases} mx+1 & ,x \leq \frac{\pi}{2} \\ \sin x+n & x > \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$, then
(a) $m=1, n=0$ (b) $m=n\frac{\pi}{2} + 1$ (c) $n=m\frac{\pi}{2}$ (d) $m=n\frac{\pi}{2}$
- The function $f(x)=\tan x$ is discontinuous on the set
(a) $\{n\pi: n \in \mathbb{Z}\}$ (b) $\{2n\pi: n \in \mathbb{Z}\}$ (c) $\{(2n+1)\frac{\pi}{2}: n \in \mathbb{Z}\}$ (d) $\{n\pi/2: n \in \mathbb{Z}\}$
- If $f(x) = x \sin\left(\frac{1}{x}\right)$, x is not zero, then the value of the function at $x=0$, so that the function is continuous at $x=0$, is
(a) 0 (b) -1 (c) 1 (d) indeterminate
- If the function $f(x) = (2x - \sin^{-1}x)/(2x + \tan^{-1}x)$ is continuous at each point of its domain, then the value of $f(0)$ is
(a) 2 (b) 1/3 (c) -1/3 (d) 2/3
- Let $f(x) = |x|$ and $g(x) = |x^3|$ then
(a) $f(x)$ and $g(x)$ both are continuous at $x=0$
(b) $f(x)$ and $g(x)$ both are differentiable at $x=0$
(c) $f(x)$ is differentiable but $g(x)$ is not differentiable at $x=0$
(d) $f(x)$ and $g(x)$ both are not differentiable at $x=0$
- The function $f(x) = 1 + |\cos x|$ is
(a) continuous no where (b) continuous everywhere
(c) not differentiable at $x=0$ (d) not differentiable at $x=n\pi, n \in \mathbb{N}$
- If $\sin(x+y) = \log(x+y)$ then $\frac{dy}{dx} =$
(a) 2 (b) -2 (c) 1 (d) -1
- The derivative of $\cos^{-1}(2x^2-1)$ with respect to $\cos^{-1}x$ is
(a) 2 (b) $1/2(1-x^2)^{1/2}$ (c) $2/x$ (d) $1-x^2$
- If $y = \log\sqrt{\tan x}$, then the value of $\frac{dy}{dx}$ at $x = \frac{\pi}{4}$ is given by
(a) ∞ (b) 1 (c) 0 (d) 1/2

ANSWERS

- 1.a 2.c 3.c 4.a 5. b 6. a 7.b 8.d 9.a 10.b

MATHEMATICS
APPLICATIONS OF DERIVATIVES
OBJECTIVE TYPE QUESTIONS



1. The rate of change of area of a circle with respect to its radius r at $r=6$ cm is
a) 10π b) 12π c) 11π d) 8π
2. Total revenue in rupees received from the sale of x units of a product is given by
 $R(x)=3x^2 + 36x + 5$. The marginal revenue, when $x=15$ is
a) Rs 116 b) Rs 96 c) Rs 36 d) Rs 126
3. If the rate of change of area of the circle is equal to the rate of change of its diameter then its radius is equal to
a) π units b) $\frac{1}{\pi}$ units c) $\frac{\pi}{2}$ units d) $\frac{2}{\pi}$ units
4. The rate of change of volume of a sphere is equal to the rate of change of the radius then its radius equal to
a) $\frac{1}{\pi}$ units b) π units c) $\frac{1}{2\sqrt{\pi}}$ units d) $\frac{1}{\sqrt{\pi}}$ units
5. The maximum value of $\sin x \cdot \cos x$ is
(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\sqrt{2}$ (D) $2\sqrt{2}$
6. The function $f(x)=\text{Sin}x(1+\text{Cos}x)$ is maximum in the interval $[0, \pi]$ at x is equal to
a) $\frac{\pi}{2}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{6}$ d) $\frac{\pi}{4}$
7. If the rate of change of volume of a sphere is equal to the rate of change of its radius then the surface area of sphere is
a) 0 b) 1 c) 3 d) $\frac{1}{3}$
8. Let f have second derivative at c such that $f'(c) = 0$ and
 $f''(c) > 0$, then c is a point of _____

ANSWERS

1. (b) 2. (d) 3. (b) 4. (c) 5. (B) 6. (b) 7. (b) 8. Local Minima



INDEFINITE INTEGRAL
OBJECTIVE TYPE QUESTIONS

1. $\int x^x(1 + \log x) dx$ is equal to
 (A) $x^x \log x + C$ (B) $\frac{x^x}{\log x} + C$ (C) $x^x + C$ (D) $x^x + 1 + C$
2. $\int \frac{dx}{1-\sin x}$ is equal to
 (A) $\tan x - \sec x + C$ (B) $\tan x + \sec x + C$ (C) $\sec x - \tan x + C$
3. $\int e^{\log \sin x} dx$ is equal to
 (A) $\sin x + C$ (B) $\cos x + C$ (C) $-\cos x + C$ (D) $-\sin x + C$
4. $\int e^x (\log \sin x + \cot x) dx$
 (A) $e^x \cot x + C$ (B) $e^x \tan x + C$ (C) $e^x \log \sin x + C$ (D) none
5. $\int \frac{\sin x}{\sin(x-a)} dx = Ax + B \log |\sin(x-a)| + C$, then the value of (A, B) is
 (A) $(\cos a, \sin a)$ (B) $(-\sin a, \cos a)$ (C) $(\sin a, \cos a)$ (D) $(-\cos a, \sin a)$
6. $\int 13^x dx = \dots\dots\dots$
 (A) $\frac{13^x}{\log x} + C$ (B) $\frac{13^x}{\log 13} + C$ (C) $\frac{13^x}{x \log 13} + C$ (D) $\frac{13^x}{13 \log x} + C$
7. $\int \frac{e^{5 \log x} + e^{4 \log x}}{e^{3 \log x} + e^{2 \log x}} dx = \dots\dots\dots$
 (A) $\frac{x^2}{2} + C$ (B) $\frac{x^3}{3} + C$ (C) $2x + C$ (D) $3x + C$
8. $\int \frac{\cos 2x - \cos 2\theta}{\cos x - \cos \theta}$ is equal to
 (A) $2(\sin x + x \cos \theta) + C$ (B) $2(\sin x - x \cos \theta) + C$
 (C) $2(\sin x + 2x \cos \theta) + C$ (D) $2(\sin x - 2x \cos \theta) + C$
9. $\int \frac{x + \sin x}{1 + \cos x} dx$ is equal to
 (A) $\log |1 + \cos x| + C$ (B) $\log |x + \sin x| + C$ (C) $x - \tan \frac{x}{2} + C$ (D) $x \cdot \tan \frac{x}{2} + C$
10. $\int e^x (\cos x - \sin x) dx$ is equal to
 (A) $e^x \cos x + C$ (B) $e^x \sin x + C$ (C) $-e^x \cos x + C$ (D) $-e^x \sin x + C$

Answers :

- (1) C (2) B (3) C (4) C (5) A (6) B (7) B (8) A (9) D (10) A

MATHEMATICS
DEFINITE INTEGRAL
OBJECTIVE TYPE QUESTIONS



1. $\int_0^{\pi/2} \frac{\sqrt{\tan x}}{\sqrt{\tan x} + \sqrt{\cot x}} dx =$

- (A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $\pi - 1$

2. $\int_{-1}^1 |1 - x| dx$ is equal to (A) 3 (B) 2 (C) -2 (D) 1

3. $\int_{-10}^{10} \log\left(\frac{2+x}{2-x}\right) dx =$ --- (A) 1 (B) 0 (C) 2 (D) e

4. $\int_0^{\pi} \log(\tan x) dx =$

- ... (A) $\frac{\pi}{4} \log \tan x$ (B) $\frac{\pi}{8} \log 2$ (C) 0 (D) $\frac{\pi}{8} \log 8$

5. $\int_{\frac{\pi}{2}}^{\pi} x^8 \sin^{-1} x dx$ is equal to (A) $\frac{5}{17}$ (B) $\frac{8}{15}$ (C) 1 (D) 0

6. If $\int_0^a f(x) dx = 10$, then $\int_0^a f(a-x) dx$, is equal to

- (A) 10 (B) -10 (C) 100 (D) 1

7. $\int_{-2}^2 |x| dx =$ --- (A) 1 (B) 2 (C) 3 (D) 4

8. $\int_0^{\frac{\pi^2}{4}} \frac{\sin \sqrt{x}}{\sqrt{x}} dx =$ --- (A) 2 (B) 1 (C) π (D) $\frac{\pi}{2}$

9. $\int_{-a}^a f(x) dx = 0$ if f is an _____ function

10. $\int_0^{\frac{\pi}{2}} \cos x e^{\sin x} dx$ is -----

ANSWERS

- (1) C (2) C (3) B (4) C (5) D (6) A (7) D (8) A (9) odd (10) e - 1



1. The area enclosed by the circle $x^2 + y^2 = 2$ is equal to -----
2. The area of the region bounded by the curve $y = x^2$ and the line $y = 16$ is ----
3. The area of the region bounded by the y -axis, $y = \cos x$ and $y = \sin x$, $0 \leq x \leq \frac{\pi}{2}$
4. The area of the region bounded by the curve $y = \sqrt{16 - x^2}$ and x -axis is -----
5. Area of the region bounded by the curve $y = \cos x$ between $x = 0$ and $x = \pi$ is
6. The area of the region bounded by parabola $y^2 = x$ and the straight line $2y = x$ is
 (A) $\frac{4}{3}$ sq units (B) 1 sq units (C) $\frac{2}{3}$ sq units (D) $\frac{1}{3}$ sq units
7. Area of the region bounded by $y^2 = 4x$, y -axis, and the line $y = 3$.
8. The area of the region bounded by the curve $x = 2y + 3$ and the y lines,
 $y = 1$ and $y = -1$ is -----
 (A) 4 sq units (B) $\frac{3}{2}$ sq units (C) 6 sq units (D) 8 sq units

Answers :

1. 2π sq units
2. $\frac{256}{3}$ sq units
3. $(\sqrt{2} - 1)$ sq units
4. 8π sq units
5. 2 sq units
6. A
7. $\frac{9}{4}$ sq units
8. C



1. The integrating factor of the differential equation $(x \log x) \frac{dy}{dx} + y = 2 \log x$

- (A) $\log(\log x)$ (B) $\log x$ (C) e^x (D) x

2. The general solution of the differential equation $\frac{dy}{dx} = \frac{y}{x}$ is

- (A) $\log y = kx$ (B) $y = kx$ (C) $xy = k$ (D) $y = k \log x$

3. The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0$,

- (A) 3 (B) 2 (C) 1 (D) not defined

4. The order of the differential equation $2x^2 \frac{d^2y}{dx^2} - 3 \frac{dy}{dx} + y = 0$, is

- (A) 2 (B) 1 (C) 0 (D) not defined

5. The number of arbitrary constants in the general solution of differential equation of fourth order is

- (A) 0 (B) 2 (C) 3 (D) 4

6. Solution of the differential equation $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$, $y(0) = 1$ is given by -----

7. Integrating factor of $\cos x \frac{dy}{dx} + y \sin x = 1$ is -----

8. Solution of the differential equation $\frac{dx}{x} + \frac{dy}{y} = 0$ is..... $xy = c$

9. The integrating factor of the differential equation $\frac{dy}{dx} + y = \frac{1+y}{x}$ is

10. The general solution of the differential equation, $\frac{dy}{dx} = e^{x-y}$ is -----

Answers

1. B 2. B 3. D 4. A 5. D 6. $\tan^{-1} y = \tan^{-1} x + \tan^{-1} 1$ 7. $\sec x$ 8. $xy = c$
 9. $\frac{e^x}{x}$ 10. $e^y = e^x + C$

VECTOR ALGEBRA
OBJECTIVE TYPE QUESTIONS

- A vector equally inclined to axes is
 a) $\hat{i} + \hat{j} + \hat{k}$ b) $\hat{i} - \hat{j} - \hat{k}$ c) $\hat{i} - \hat{j} + \hat{k}$ d) $-\hat{i} + \hat{j} + \hat{k}$
- If the vectors are equal then their magnitudes are equal but the converse may not be true. State true or false
- If $\vec{a} = \hat{i} + 2\hat{j} - \hat{k}$, $\vec{b} = 3\hat{i} + \hat{j} - 5\hat{k}$. Find a unit vector in the direction of $\hat{a} - \hat{b}$.
- For what value of “a” the vectors $2\hat{i} - 3\hat{j} + 4\hat{k}$ and $a\hat{i} + 6\hat{j} - 8\hat{k}$ are collinear
- If $|\vec{a}| = 5$, $|\vec{b}| = 13$, $|\vec{a} \times \vec{b}| = 25$ find $\vec{a} \cdot \vec{b}$
- Find the projection of the vector $\hat{i} + 3\hat{j} + 7\hat{k}$ on the vector $7\hat{i} - \hat{j} + 8\hat{k}$
- Find the area of a parallelogram whose adjacent sides are given by the vectors $\vec{a} = 3\hat{i} + \hat{j} + 4\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$
- For what value of p the vectors $\vec{a} = 2\hat{i} + p\hat{j} + \hat{k}$ and $\hat{i} - 2\hat{j} + 3\hat{k}$ are perpendicular to each other
 a) 5 b) -2 c) $\frac{5}{2}$ d) 3
- If $|\vec{a}| = 4$ and $-3 \leq \lambda \leq 2$ then the range of $|\lambda\vec{a}|$ is
 a) [0,8] b) [-12,8] c) [0,12] d) [8,12]
- If $\vec{a} = \hat{i} + \hat{j}$; $\vec{b} = \hat{j} + \hat{k}$; $\vec{c} = \hat{k} + \hat{i}$, find a unit vector in the direction of $2\vec{a} + \vec{b} + \vec{c}$.

ANSWERS

- 1)a 2). True 3). $\frac{-2\hat{i} + \hat{j} + 4\hat{k}}{\sqrt{21}}$ 4). -4 5) 60 6) $\frac{60}{\sqrt{114}}$
 7) $\sqrt{42}$ 8) $\frac{5}{2}$ 9) a 10) $\frac{3\hat{i} + 3\hat{j} + 2\hat{k}}{\sqrt{22}}$

3 DIMENSIONAL GEOMETRY
MULTIPLE CHOICE QUESTIONS

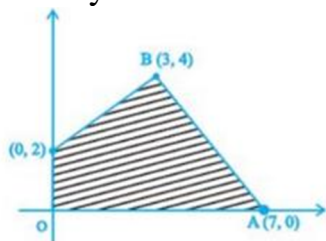
- If a line makes angles 90° , 60° and θ with x, y and z axes respectively, where θ is acute then the value of θ is
[(A) 30° (B) 60° (C) 90° (D) 45°]
- The distance between the planes $2x + 2y - z + 2 = 0$ and $4x + 4y - 2z + s = 0$ is
[(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{1}{6}$ (D) 1]
- The acute angle between the planes $2x - y + z = 6$ and $x + y + 2z = 3$ is
[(A) 45° (B) 60° (C) 30° (D) 75°]
- If a line makes angles α, β, γ with the axes then $\cos 2\alpha + \cos 2\beta + \cos 2\gamma$ is equal to
[(A) -2 (B) -1 (C) 1 (D) 2]
- The angle between the line $\vec{r} = 2\hat{i} - \hat{j} + \hat{k} + \lambda(-\hat{i} + \hat{j} + \hat{k})$ and the plane $(3\hat{i} + 2\hat{j} - \hat{k}) \cdot \vec{r} = 4$
[(A) $\cos^{-1}\left(\frac{2}{\sqrt{42}}\right)$ (B) $\cos^{-1}\left(\frac{-2}{\sqrt{42}}\right)$ (C) $\sin^{-1}\left(\frac{2}{\sqrt{42}}\right)$ (D) $\sin^{-1}\left(\frac{-2}{\sqrt{42}}\right)$]
- If the lines $\frac{x-2}{1} = \frac{y-2}{1} = \frac{z-4}{k}$ and $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$ are coplanar, then k can have
[(A) Any value (B) Exactly one value (C) Exactly two values (D) Exactly three values]
- The equation of the plane which cuts equal intercepts of unit length on the coordinate axes is
[(A) $x + y + z = 1$ (B) $x + y + z = 0$ (C) $x + y - z = 1$ (D) $x + y + z = 2$]
- If the lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect at a point, then the value of k is
[(A) $\frac{3}{2}$ (B) $\frac{9}{2}$ (C) $\frac{2}{9}$ (D) 2]
- The direction cosines of the normal to the plane $x + 2y - 3z - 4 = 0$ are
[(A) $\frac{-1}{\sqrt{14}}, \frac{-2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$ (B) $\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$ (C) $\frac{-1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$ (D) $\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$]
- The angle between two diagonals of a cube is
[(A) $\cos^{-1}\frac{1}{\sqrt{3}}$ (B) $\cos^{-1}\frac{1}{3}$ (C) $\cos^{-1}\frac{1}{9}$ (D) $\cos^{-1}\frac{\sqrt{3}}{2}$]

Answers :

- A
- C
- B
- B
- D
- C
- A
- B
- D
- B

LINEAR PROGRAMMING
MULTIPLE CHOICE QUESTIONS

- In maximization problem, optimal solution occurring at corner point yields the
 - mean values of z
 - highest value of z
 - lowest value of z
 - mid values of z
- In a LPP, the objective function is always
 - Linear
 - Quadratic
 - Cubic
 - Biquadratic
- A feasible solution to a linear programming problem
 - Must satisfy all of the problem's constraints simultaneously
 - Need not satisfy all of the constraints, only the non-negativity constraints
 - Must be a corner point of the feasible region
 - Must give the maximum possible profit
- The corner points of the bounded feasible region of a LPP are $A(0,50)$, $B(20, 40)$, $C(50, 100)$ and $D(0, 200)$ and the objective function is $Z = x + 2y$. Then the maximum value is
 - 100
 - 400
 - 250
 - 450
- The feasible region (shaded) for a L.P.P is shown in the figure. The maximum $Z = 5x + 7y$ is



[A. 49 B. 45 C. 43 D. 47]

Answers :

1. B 2. A 3. A 4. B 5. C

PROBABILITY
OBJECTIVE TYPE QUESTIONS

- If A and B are independent events the $P(A \cap B) = \dots\dots$
- If a fair die is rolling. The events are $E = \{1, 3, 6\}$, $F = \{4, 6\}$. Then the probability $P(E/F)$ is...
A. $\frac{1}{6}$ B. $\frac{1}{3}$ C. $\frac{1}{2}$ D. $\frac{2}{3}$
- Let A and B are two events. If $P(A) = 0.2$, $P(B) = 0.4$, $P(A \cup B) = 0.6$, then $P(A/B)$ is equal to ...
A. 0.8 B. 0.5 C. 0.3 D. 0
- Let A and B be two events such that $P(A) = 0.6$, $P(B) = 0.2$ and $P(A/B) = 0.5$, then $P(A' / B')$ equals...
A. $\frac{1}{10}$ B. $\frac{3}{10}$ C. $\frac{3}{8}$ D. $\frac{6}{7}$
- Two numbers are chosen from $\{1, 2, 3, 4, 5, 6\}$ one after the other without replacement. The probability that one of the smaller values is less than 4 is
A. $\frac{4}{5}$ B. $\frac{1}{15}$ C. $\frac{1}{5}$ D. $\frac{14}{15}$
- The probability of a student getting 1, 2, 3 division in an examination are $\frac{1}{10}$, $\frac{3}{5}$ and $\frac{1}{4}$ respectively. The probability that the student fails in the examination is
A. $\frac{197}{200}$ B. $\frac{27}{100}$ C. $\frac{83}{100}$ D. None of these
- A speaks truth in 75% cases and B speaks truth in 80% cases. The probability that they contradict each other in a statement is
A. $\frac{7}{20}$ B. $\frac{13}{20}$ C. $\frac{3}{5}$ D. $\frac{2}{5}$
- The probability that a leap year will have 53 fridays or 53 saturdays
A. 2/7 B. 3/7 C. 4/7 D. 1/7
- A person writes 4 letters and addresses 4 envelopes. If the letters are placed in the envelopes at random, then the probability that all letters are not placed in the right envelopes, is
A. 1/4 B. 11/24 C. 15/24 D. 23/24

ANSWERS

- | | |
|----------------------|------|
| 1. $P(A) \cdot P(B)$ | 5. A |
| 2. C | 6. B |
| 3. D | 7. A |
| 4. C | 8. B |
| | 9. D |

**SET-1****Series BVM/C**कोड नं. **65/1/1**
Code No.रोल नं.

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Roll No.

परीक्षार्थी कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Code on the title page of the answer-book.

- कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **11** हैं ।
- प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए कोड नम्बर को छात्र उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।
- कृपया जाँच कर लें कि इस प्रश्न-पत्र में **29** प्रश्न हैं ।
- कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, प्रश्न का क्रमांक अवश्य लिखें ।
- इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।
- Please check that this question paper contains **11** printed pages.
- Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains **29** questions.
- **Please write down the Serial Number of the question before attempting it.**
- 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

गणित**MATHEMATICS**

निर्धारित समय : 3 घण्टे

अधिकतम अंक : 100

Time allowed : 3 hours

Maximum Marks : 100



सामान्य निर्देश:

- सभी प्रश्न अनिवार्य हैं।
- इस प्रश्न-पत्र में 29 प्रश्न हैं जो चार खण्डों में विभाजित हैं: अ, ब, स तथा द। खण्ड अ में 4 प्रश्न हैं जिनमें से प्रत्येक एक अंक का है। खण्ड ब में 8 प्रश्न हैं जिनमें से प्रत्येक दो अंक का है। खण्ड स में 11 प्रश्न हैं जिनमें से प्रत्येक चार अंक का है। खण्ड द में 6 प्रश्न हैं जिनमें से प्रत्येक छः अंक का है।
- खण्ड अ में सभी प्रश्नों के उत्तर एक शब्द, एक वाक्य अथवा प्रश्न की आवश्यकतानुसार दिए जा सकते हैं।
- पूर्ण प्रश्न-पत्र में विकल्प नहीं हैं। फिर भी खण्ड अ के 1 प्रश्न में, खण्ड ब के 3 प्रश्नों में, खण्ड स के 3 प्रश्नों में तथा खण्ड द के 3 प्रश्नों में आन्तरिक विकल्प हैं। ऐसे सभी प्रश्नों में से आपको एक ही विकल्प हल करना है।
- कैलकुलेटर के प्रयोग की अनुमति नहीं है। यदि आवश्यक हो, तो आप लघुगणकीय सारणियाँ माँग सकते हैं।

General Instructions :

- All questions are compulsory.
- The question paper consists of 29 questions divided into four sections A, B, C and D. Section A comprises of 4 questions of **one mark** each, Section B comprises of 8 questions of **two marks** each, Section C comprises of 11 questions of **four marks** each and Section D comprises of 6 questions of **six marks** each.
- All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
- There is no overall choice. However, internal choice has been provided in 1 question of Section A, 3 questions of Section B, 3 questions of Section C and 3 questions of Section D. You have to attempt only one of the alternatives in all such questions.
- Use of calculators is **not** permitted. You may ask for logarithmic tables, if required.

खण्ड अ

SECTION A

प्रश्न संख्या 1 से 4 तक प्रत्येक प्रश्न 1 अंक का है।

Question numbers 1 to 4 carry 1 mark each.

1. सारणिक $\begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$ के अवयव a_{23} का सहखण्ड ज्ञात कीजिए।

Find the cofactor of the element a_{23} of the determinant $\begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$.

2. x के सापेक्ष $\cos \{\sin(x)^2\}$ का अवकलन कीजिए।

Differentiate $\cos \{\sin(x)^2\}$ with respect to x .



3. निम्न अवकल समीकरण की कोटि व घात ज्ञात कीजिए :

$$\left(\frac{dy}{dx}\right)^3 + 2y \frac{d^2y}{dx^2} = 0$$

Determine the order and the degree of the differential equation

$$\left(\frac{dy}{dx}\right)^3 + 2y \frac{d^2y}{dx^2} = 0$$

4. z-अक्ष पर, बिन्दु P(3, -4, 5) से डाले गए लम्ब की लम्बाई ज्ञात कीजिए ।

अथवा

एक समतल का सदिश समीकरण ज्ञात कीजिए जो मूल-बिन्दु से 5 इकाई की दूरी पर है तथा जिसका अभिलंब सदिश $2\hat{i} - \hat{j} + 2\hat{k}$ है ।

Find the length of the perpendicular drawn from the point P(3, -4, 5) on the z-axis.

OR

Find the vector equation of a plane, which is at a distance of 5 units from the origin and whose normal vector is $2\hat{i} - \hat{j} + 2\hat{k}$.

खण्ड ब

SECTION B

प्रश्न संख्या 5 से 12 तक प्रत्येक प्रश्न के 2 अंक हैं ।

Question numbers 5 to 12 carry 2 marks each.

5. यदि $f: \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = (3 - x^3)^{\frac{1}{3}}$ द्वारा परिभाषित है, तो $f \circ f(x)$ ज्ञात कीजिए ।

If $f: \mathbb{R} \rightarrow \mathbb{R}$ is given by $f(x) = (3 - x^3)^{\frac{1}{3}}$, find $f \circ f(x)$.

6. यदि $A = \begin{bmatrix} 5 & -3 \\ -3 & 2 \end{bmatrix}$ तथा $B^{-1} = \begin{bmatrix} 3 & 2 \\ 0 & -1 \end{bmatrix}$ है, तो A^{-1} ज्ञात कीजिए । अतः

$(AB)^{-1}$ भी ज्ञात कीजिए ।

If $A = \begin{bmatrix} 5 & -3 \\ -3 & 2 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 3 & 2 \\ 0 & -1 \end{bmatrix}$, find A^{-1} and hence find $(AB)^{-1}$.

7. ज्ञात कीजिए :

$$\int \frac{dx}{\sqrt{2x - x^2}}$$



Find :

$$\int \frac{dx}{\sqrt{2x - x^2}}$$

8. ज्ञात कीजिए :

$$\int \frac{1}{\cos^2 x (1 - \tan x)^2} dx$$

अथवा

मान ज्ञात कीजिए :

$$\int_0^1 x (1 - x)^n dx$$

Find :

$$\int \frac{1}{\cos^2 x (1 - \tan x)^2} dx$$

OR

Evaluate :

$$\int_0^1 x (1 - x)^n dx$$

9. वक्रों के कुल $y = b \cos (x + a)$, जहाँ a और b स्वेच्छ अचर हैं, को निरूपित करने वाले अवकल समीकरण को ज्ञात कीजिए ।

Form the differential equation representing the family of curves $y = b \cos (x + a)$, where a and b are arbitrary constants.

10. मात्रक सदिश ज्ञात कीजिए जो सदिश \vec{a} और \vec{b} दोनों पर लंब हो, जहाँ

$$\vec{a} = 4\hat{i} - \hat{j} + 8\hat{k}; \vec{b} = -\hat{j} + \hat{k}.$$

अथवा

यदि $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ तथा $\vec{c} = 3\hat{i} + \hat{j}$ ऐसे सदिश हैं कि $\vec{a} + \lambda \vec{b}$ सदिश \vec{c} पर लंब है, तो λ का मान ज्ञात कीजिए ।



Find a unit vector perpendicular to both \vec{a} and \vec{b} , where

$$\vec{a} = 4\hat{i} - \hat{j} + 8\hat{k}, \vec{b} = -\hat{j} + \hat{k}.$$

OR

If $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$ are such that $\vec{a} + \lambda \vec{b}$ is perpendicular to \vec{c} , then find the value of λ .

11. A और B दो स्वतंत्र घटनाएँ हैं जिनके लिए $P(A) = 0.3$ और $P(B) = 0.5$ है। $P(A|B)$ ज्ञात कीजिए।

अथवा

एक थैले में 3 सफेद व 2 लाल गेंदें हैं और दूसरे थैले में 4 सफेद व 3 लाल गेंदें हैं। प्रत्येक थैले से एक गेंद यादृच्छया निकाली जाती है। निकाली गई गेंदों में एक सफेद व एक लाल गेंद होने की प्रायिकता ज्ञात कीजिए।

Given that A and B are two independent events such that $P(A) = 0.3$ and $P(B) = 0.5$. Find $P(A|B)$.

OR

A bag contains 3 white and 2 red balls, another bag contains 4 white and 3 red balls. One ball is drawn at random from each bag. Find the probability that the balls drawn are one white and one red.

12. एक प्रश्न को A, B और C द्वारा स्वतंत्र रूप से हल करने की प्रायिकताएँ क्रमशः $\frac{1}{2}$, $\frac{1}{3}$ और $\frac{1}{4}$ हैं। यदि तीनों, स्वतंत्र रूप से, प्रश्न हल करने का प्रयास करते हैं, तो प्रश्न के हल हो जाने की प्रायिकता ज्ञात कीजिए।

The probabilities of A, B and C solving a problem independently are $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ respectively. If all the three try to solve the problem independently, find the probability that the problem is solved.



खण्ड स
SECTION C

प्रश्न संख्या 13 से 23 तक प्रत्येक प्रश्न के 4 अंक हैं।

Question numbers 13 to 23 carry 4 marks each.

13. मान लीजिए कि $A = \mathbf{N} \times \mathbf{N}$ सभी प्राकृत संख्याओं के क्रमित युग्मों का समुच्चय है और संबंध R समुच्चय A पर $(a, b) R (c, d)$ यदि व केवल यदि $ad = bc$ द्वारा परिभाषित है। दर्शाइए कि R एक तुल्यता संबंध है।

अथवा

दर्शाइए कि $f: \mathbf{R} - \{2\} \rightarrow \mathbf{R} - \{1\}$, $f(x) = \frac{x}{x-2}$ द्वारा परिभाषित फलन एकैकी है। यदि $g: \mathbf{R} - \{1\} \rightarrow \mathbf{R} - \{2\}$, $g(x) = \frac{2x}{x-1}$ द्वारा परिभाषित है, तो $\text{gof}(x)$ ज्ञात कीजिए।

Let $A = \mathbf{N} \times \mathbf{N}$ be the set of all ordered pairs of natural numbers and R be the relation on the set A defined by $(a, b) R (c, d)$ iff $ad = bc$. Show that R is an equivalence relation.

OR

Show that $f: \mathbf{R} - \{2\} \rightarrow \mathbf{R} - \{1\}$ defined by $f(x) = \frac{x}{x-2}$ is one-one. Also, if $g: \mathbf{R} - \{1\} \rightarrow \mathbf{R} - \{2\}$ is defined as $g(x) = \frac{2x}{x-1}$, find $\text{gof}(x)$.

14. सिद्ध कीजिए कि :

$$\tan^{-1}\left(\frac{\sqrt{1+x} + \sqrt{1-x}}{\sqrt{1+x} - \sqrt{1-x}}\right) = \frac{\pi}{4} + \frac{1}{2} \cos^{-1} x; \quad \frac{-1}{\sqrt{2}} \leq x \leq 1$$

Prove that :

$$\tan^{-1}\left(\frac{\sqrt{1+x} + \sqrt{1-x}}{\sqrt{1+x} - \sqrt{1-x}}\right) = \frac{\pi}{4} + \frac{1}{2} \cos^{-1} x; \quad \frac{-1}{\sqrt{2}} \leq x \leq 1$$

15. सारणिकों के गुणधर्मों का प्रयोग करके, सिद्ध कीजिए कि

$$\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = (x-y)(y-z)(z-x)$$

Using properties of determinants, show that

$$\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = (x-y)(y-z)(z-x)$$



16. यदि $x^y \cdot y^x = x^x$ है, तो $\frac{dy}{dx}$ ज्ञात कीजिए ।
अथवा

यदि $x = a \sec^3 \theta$ तथा $y = a \tan^3 \theta$ है, तो $\frac{d^2y}{dx^2}$ ज्ञात कीजिए ।

Find $\frac{dy}{dx}$, if $x^y \cdot y^x = x^x$.

OR

If $x = a \sec^3 \theta$ and $y = a \tan^3 \theta$, find $\frac{d^2y}{dx^2}$.

17. यदि $y = a \cos(\log x) + b \sin(\log x)$ है, तो दर्शाइए कि $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$.

If $y = a \cos(\log x) + b \sin(\log x)$, show that $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$.

18. वक्र $ay^2 = x^3$ के बिन्दु (am^2, am^3) पर स्पर्श-रेखा का समीकरण ज्ञात कीजिए ।

Find the equation of the tangent to the curve $ay^2 = x^3$ at the point (am^2, am^3) .

19. ज्ञात कीजिए :

$$\int \frac{\cos x}{(1 + \sin x)(2 + \sin x)} dx$$

Find :

$$\int \frac{\cos x}{(1 + \sin x)(2 + \sin x)} dx$$

20. मान ज्ञात कीजिए :

$$\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx$$

Evaluate :

$$\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx$$

21. अवकल समीकरण $x dx - y e^y \sqrt{1+x^2} dy = 0$ का विशिष्ट हल ज्ञात कीजिए, दिया गया है $y = 1$ जबकि $x = 0$.

अथवा



अवकल समीकरण $x \cos\left(\frac{y}{x}\right) \frac{dy}{dx} = y \cos\left(\frac{y}{x}\right) + x$ को हल कीजिए ।

Find the particular solution of the differential equation

$x dx - y e^y \sqrt{1+x^2} dy = 0$, given that $y = 1$ when $x = 0$.

OR

Solve the differential equation $x \cos\left(\frac{y}{x}\right) \frac{dy}{dx} = y \cos\left(\frac{y}{x}\right) + x$.

22. दर्शाइए कि चार बिन्दु A, B, C तथा D जिनके स्थिति सदिश क्रमशः $\hat{i} + 2\hat{j} - \hat{k}$, $3\hat{i} - \hat{j}$, $2\hat{i} + 3\hat{j} + 2\hat{k}$ तथा $4\hat{i} + 3\hat{k}$ समतलीय हैं ।

Show that the four points A, B, C and D with position vectors $\hat{i} + 2\hat{j} - \hat{k}$, $3\hat{i} - \hat{j}$, $2\hat{i} + 3\hat{j} + 2\hat{k}$ and $4\hat{i} + 3\hat{k}$ respectively are coplanar.

23. समतलों $\vec{r} \cdot (3\hat{i} + 4\hat{j} + 2\hat{k}) = 5$ एवं $\vec{r} \cdot (3\hat{i} - 2\hat{j} - 2\hat{k}) = 4$ के समान्तर उस रेखा का सदिश समीकरण ज्ञात कीजिए जो बिन्दु $(2, 3, -1)$ से गुज़रती है ।

Find the vector equation of the line passing through the point $(2, 3, -1)$ and parallel to the planes $\vec{r} \cdot (3\hat{i} + 4\hat{j} + 2\hat{k}) = 5$ and $\vec{r} \cdot (3\hat{i} - 2\hat{j} - 2\hat{k}) = 4$.

खण्ड द

SECTION D

प्रश्न संख्या 24 से 29 तक प्रत्येक प्रश्न के 6 अंक हैं ।

Question numbers 24 to 29 carry 6 marks each.

24. यदि $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ है, तो A^{-1} ज्ञात कीजिए । A^{-1} का प्रयोग करके, निम्न समीकरण

निकाय को हल कीजिए :

$$y + 2z = 5$$

$$x + 2y + 3z = 10$$

$$3x + y + z = 9$$

अथवा



प्रारम्भिक संक्रियाओं का प्रयोग करके, आव्यूह $A = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ का व्युत्क्रम

प्राप्त कीजिए ।

If $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$, find A^{-1} . Using A^{-1} , solve the system of equations

$$y + 2z = 5$$

$$x + 2y + 3z = 10$$

$$3x + y + z = 9$$

OR

Obtain the inverse of the following matrix using elementary operations :

$$A = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

25. वृत्त $x^2 + y^2 = 80$ का वह बिन्दु ज्ञात कीजिए जो बिन्दु $(1, 2)$ से न्यूनतम दूरी पर है ।
Find the point on the circle $x^2 + y^2 = 80$ which is nearest to the point $(1, 2)$.
26. समाकलन विधि से दो वृत्तों $x^2 + y^2 = 1$ एवं $(x - 1)^2 + y^2 = 1$ के बीच घिरे क्षेत्र का क्षेत्रफल ज्ञात कीजिए ।

अथवा

समाकलन विधि से, निम्न क्षेत्र का क्षेत्रफल ज्ञात कीजिए :

$$\{(x, y) : 9x^2 + 4y^2 \leq 36, 3x + 2y \geq 6\}$$

Using integration, find the area of the region enclosed between the two circles $x^2 + y^2 = 1$ and $(x - 1)^2 + y^2 = 1$.

OR

Using integration, find the area of the region :

$$\{(x, y) : 9x^2 + 4y^2 \leq 36, 3x + 2y \geq 6\}$$



27. रेखा $\frac{x-8}{4} = \frac{y-1}{1} = \frac{z-3}{8}$ समतल $2x + 2y + z = 3$ को जिस बिन्दु पर काटती है, उसके निर्देशांक ज्ञात कीजिए। रेखा एवं समतल के बीच का कोण भी ज्ञात कीजिए।

अथवा

दिक्-अनुपात $\langle 2, 2, 1 \rangle$ वाली रेखा निम्न दो रेखाओं को क्रमशः बिन्दु P तथा Q पर काटती है :

$$\frac{x-7}{3} = \frac{y-5}{2} = \frac{z-3}{1} \text{ तथा } \frac{x-1}{2} = \frac{y+1}{4} = \frac{z+1}{3}$$

अन्तःखण्ड PQ का समीकरण एवं लम्बाई ज्ञात कीजिए।

Find the coordinates of the point where the line $\frac{x-8}{4} = \frac{y-1}{1} = \frac{z-3}{8}$ intersects the plane $2x + 2y + z = 3$. Also find the angle between the line and the plane.

OR

A line with direction ratios $\langle 2, 2, 1 \rangle$ intersects the lines

$$\frac{x-7}{3} = \frac{y-5}{2} = \frac{z-3}{1} \text{ and } \frac{x-1}{2} = \frac{y+1}{4} = \frac{z+1}{3}$$

at the points P and Q respectively. Find the length and the equation of the intercept PQ.

28. एक छोटी फर्म कुर्सियाँ और मेजें बनाती है। बाज़ार की माँग व उपलब्ध साधनों के अनुसार प्रतिदिन कुर्सियाँ और मेजों का कुल उत्पादन 50 इकाइयों से अधिक नहीं होना चाहिए। एक कुर्सी को बनाने में 30 मिनट व एक मेज़ को बनाने में 1 घंटा लगता है। प्रतिदिन अधिकतम 40 कार्य घंटे उपलब्ध हैं। एक कुर्सी पर ₹ 40 का लाभ व एक मेज़ पर ₹ 60 का लाभ होता है। ज्ञात कीजिए, अधिकतम लाभ अर्जित करने के लिए प्रतिदिन कितनी कुर्सियाँ और कितनी मेजें बनाई जाएँ। अधिकतम लाभ भी ज्ञात कीजिए। रैखिक प्रोग्रामन समस्या बनाकर उसे आलेख विधि से हल कीजिए।

A small firm manufactures chairs and tables. Market demand and available resources indicate that the combined production of chairs and tables should not exceed 50 units per day. It takes 30 minutes to manufacture a chair and 1 hour to manufacture a table. A maximum of 40 man-hours per day are available. The profit on each chair is ₹ 40 and profit on each table is ₹ 60. Determine how many each of chairs and tables should be manufactured per day in order to maximize the profit. What is the maximum profit? Formulate LPP and solve graphically.



29. थैले I में, 4 लाल गेंदें व 2 हरी गेंदें हैं और थैले II में, 3 लाल गेंदें व 5 हरी गेंदें हैं। थैले I से यादृच्छया एक गेंद निकाल कर थैले II में डाली जाती है और तत्पश्चात् थैले II से यादृच्छया एक गेंद निकाली जाती है। निकाली गई गेंद हरे रंग की है। प्रायिकता ज्ञात कीजिए कि स्थानांतरित की गई गेंद भी हरी थी।

Bag I contains 4 red and 2 green balls and Bag II contains 3 red and 5 green balls. One ball is transferred at random from Bag I to Bag II and then a ball is drawn at random from Bag II. The ball so drawn is found to be green in colour. Find the probability that the transferred ball is also green.

**SET – 1****Series : BVM/1****कोड नं. 65/1/1**
Code No.**रोल नं.**

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Roll No.

परीक्षार्थी कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।

Candidates must write the Code on the title page of the answer-book.

- कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ 11 हैं ।
- प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए कोड नम्बर को छात्र उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।
- कृपया जाँच कर लें कि इस प्रश्न-पत्र में 29 प्रश्न हैं ।
- कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, प्रश्न का क्रमांक अवश्य लिखें ।
- इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।
- Please check that this question paper contains 11 printed pages.
- Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains 29 questions.
- **Please write down the Serial Number of the question before attempting it.**
- 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

गणित**MATHEMATICS**

निर्धारित समय : 3 घण्टे

Time allowed : 3 hours

अधिकतम अंक : 100

Maximum Marks : 100

65/1/1

1

[P.T.O.]

सामान्य निर्देश :

- (i) सभी प्रश्न अनिवार्य हैं।
- (ii) इस प्रश्न-पत्र में 29 प्रश्न हैं जो चार खण्डों में विभाजित हैं : अ, ब, स तथा द। खण्ड अ में 4 प्रश्न हैं जिनमें से प्रत्येक एक अंक का है। खण्ड ब में 8 प्रश्न हैं जिनमें से प्रत्येक दो अंक का है। खण्ड स में 11 प्रश्न हैं जिनमें से प्रत्येक चार अंक का है। खण्ड द में 6 प्रश्न हैं जिनमें से प्रत्येक छः अंक का है।
- (iii) खण्ड अ में सभी प्रश्नों के उत्तर एक शब्द, एक वाक्य अथवा प्रश्न की आवश्यकतानुसार दिए जा सकते हैं।
- (iv) पूर्ण प्रश्न-पत्र में विकल्प नहीं हैं। फिर भी खण्ड अ के 1 प्रश्न, खण्ड ब के 3 प्रश्नों में, खण्ड स के 3 प्रश्नों में तथा खण्ड द के 3 प्रश्नों में आंतरिक विकल्प हैं। ऐसे सभी प्रश्नों में, से आपको एक ही विकल्प हल करना है।
- (v) कैलकुलेटर के प्रयोग की अनुमति नहीं है। यदि आवश्यक हो, तो आप लघुगणकीय सारणियाँ माँग सकते हैं।

General Instructions :

- (i) All questions are compulsory.
- (ii) This question paper contains 29 questions divided into four sections A, B, C and D. Section A comprises of 4 questions of **one mark** each, Section B comprises of 8 questions of **two marks** each, Section C comprises of 11 questions of **four marks** each and Section D comprises of 6 questions of **six marks** each.
- (iii) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
- (iv) There is no overall choice. However, internal choice has been provided in 1 question of Section A, 3 questions of Section B, 3 questions of Section C and 3 questions of Section D. You have to attempt only **one** of the alternatives in all such questions.
- (v) Use of calculators is not permitted. You may ask logarithmic tables, if required.

खण्ड – अ

SECTION – A

प्रश्न संख्या 1 से 4 तक के प्रत्येक प्रश्न 1 अंक का है।

Question numbers 1 to 4 carry 1 mark each.

1. यदि A और B एक ही कोटि 3 के वर्ग आव्यूह हैं और $|A| = 2$ तथा $AB = 2I$ है, तो $|B|$ का मान लिखिए।

If A and B are square matrices of the same order 3, such that $|A| = 2$ and $AB = 2I$, write the value of $|B|$.



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2. यदि $f(x) = x + 1$ है, तो $\frac{d}{dx} (f \circ f)(x)$ ज्ञात कीजिए।

If $f(x) = x + 1$, find $\frac{d}{dx} (f \circ f)(x)$.

3. अवकल समीकरण $x^2 \frac{d^2y}{dx^2} = \left\{ 1 + \left(\frac{dy}{dx} \right)^2 \right\}^4$ की कोटि व घात ज्ञात कीजिए।

Find the order and the degree of the differential equation $x^2 \frac{d^2y}{dx^2} = \left\{ 1 + \left(\frac{dy}{dx} \right)^2 \right\}^4$.

4. यदि एक रेखा x -अक्ष, y -अक्ष तथा z -अक्ष से क्रमशः 90° , 135° , 45° के कोण बनाती है। इस रेखा के दिक्-कोसाइन ज्ञात कीजिए।

अथवा

उस रेखा का सदिश समीकरण ज्ञात कीजिए जो बिन्दु $(3, 4, 5)$ से गुजरती है तथा सदिश $2\hat{i} + 2\hat{j} - 3\hat{k}$ के समांतर है।

If a line makes angles 90° , 135° , 45° with the x , y and z axes respectively, find its direction cosines.

OR

Find the vector equation of the line which passes through the point $(3, 4, 5)$ and is parallel to the vector $2\hat{i} + 2\hat{j} - 3\hat{k}$.

खण्ड – ब

SECTION – B

प्रश्न संख्या 5 से 12 तक के प्रत्येक प्रश्न के 2 अंक हैं।

Question numbers 5 to 12 carry 2 marks each.

5. जाँच कीजिए कि क्या संक्रिया $*$ जो R पर $a * b = ab + 1$ द्वारा परिभाषित है (i) द्वि-आधारी संक्रिया होगी या नहीं (ii) यदि यह द्वि-आधारी है, तो क्या यह साहचर्य होगी या नहीं ?

Examine whether the operation $*$ defined on R by $a * b = ab + 1$ is (i) a binary or not. (ii) if a binary operation, is it associative or not ?



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6. आव्यूह A ज्ञात कीजिए यदि $2A - 3B + 5C = O$, जहाँ $B = \begin{bmatrix} -2 & 2 & 0 \\ 3 & 1 & 4 \end{bmatrix}$ तथा $C = \begin{bmatrix} 2 & 0 & -2 \\ 7 & 1 & 6 \end{bmatrix}$ ।

Find a matrix A such that $2A - 3B + 5C = O$, where $B = \begin{bmatrix} -2 & 2 & 0 \\ 3 & 1 & 4 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & 0 & -2 \\ 7 & 1 & 6 \end{bmatrix}$.

7. ज्ञात कीजिए : $\int \frac{\sec^2 x}{\sqrt{\tan^2 x + 4}} dx$.

Find : $\int \frac{\sec^2 x}{\sqrt{\tan^2 x + 4}} dx$.

8. ज्ञात कीजिए : $\int \sqrt{1 - \sin 2x} dx, \frac{\pi}{4} < x < \frac{\pi}{2}$

अथवा

ज्ञात कीजिए : $\int \sin^{-1}(2x) dx$.

Find : $\int \sqrt{1 - \sin 2x} dx, \frac{\pi}{4} < x < \frac{\pi}{2}$

OR

Find : $\int \sin^{-1}(2x) dx$.

9. वक्रों के कुल $y = e^{2x}(a + bx)$, जिसमें a, b स्वेच्छ अचर हैं, को निरूपित करने वाला अवकल समीकरण ज्ञात कीजिए।

Form the differential equation representing the family of curves $y = e^{2x}(a + bx)$, where 'a' and 'b' are arbitrary constants.



10. यदि दो मात्रक सदिशों का योग एक मात्रक सदिश हो, तो सिद्ध कीजिए कि उन दो सदिशों के अन्तर का परिमाण $\sqrt{3}$ होगा।

अथवा

यदि $\vec{a} = 2\hat{i} + 3\hat{j} + \hat{k}$, $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$ तथा $\vec{c} = -3\hat{i} + \hat{j} + 2\hat{k}$ है, तो $[\vec{a} \vec{b} \vec{c}]$ ज्ञात कीजिए।

If the sum of two unit vectors is a unit vector, prove that the magnitude of their difference is $\sqrt{3}$.

OR

If $\vec{a} = 2\hat{i} + 3\hat{j} + \hat{k}$, $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$ and $\vec{c} = -3\hat{i} + \hat{j} + 2\hat{k}$, find $[\vec{a} \vec{b} \vec{c}]$.

11. एक पाँसा जिस पर 1, 2, 3 लाल रंग से तथा 4, 5, 6 हरे रंग से लिखा गया है, को उछाला जाता है। “संख्या सम होने” की घटना को A से व “संख्या लाल रंग में लिखी है” की घटना B से परिभाषित है। ज्ञात कीजिए कि क्या ये दो घटनाएँ A तथा B स्वतंत्र हैं या नहीं।

A die marked 1, 2, 3 in red and 4, 5, 6 in green is tossed. Let A be the event “number is even” and B be the event “number is marked red”. Find whether the events A and B are independent or not.

12. एक पासे को छः बार उछाला जाता है। यदि “पासे पर विषम संख्या प्राप्त होना” एक सफलता है, तो (i) 5 सफलताएँ (ii) अधिकतम 5 सफलताएँ, की प्रायिकताएँ क्या-क्या होंगी ?

अथवा

एक यादृच्छिक चर X का प्रायिकता बंटन P(X) निम्न प्रकार से है, जहाँ ‘k’ कोई संख्या है :

$$P(X = x) = \begin{cases} k, & \text{यदि } x = 0 \\ 2k, & \text{यदि } x = 1 \\ 3k, & \text{यदि } x = 2 \\ 0, & \text{अन्यथा} \end{cases}$$

‘k’ का मान ज्ञात कीजिए।

A die is thrown 6 times. If “getting an odd number” is a “success”, what is the probability of (i) 5 successes ? (ii) atmost 5 successes ?

OR

The random variable X has a probability distribution P(X) of the following form, where ‘k’ is some number.

$$P(X = x) = \begin{cases} k, & \text{if } x = 0 \\ 2k, & \text{if } x = 1 \\ 3k, & \text{if } x = 2 \\ 0, & \text{otherwise} \end{cases}$$

Determine the value of ‘k’.



खण्ड – स

SECTION – C

प्रश्न संख्या 13 से 23 के प्रत्येक प्रश्न के 4 अंक हैं।

Question numbers 13 to 23 carry 4 marks each.

13. दिखाइए कि समुच्चय \mathbb{R} में $R = \{(a, b) : a \leq b\}$ द्वारा परिभाषित संबंध R स्वतुल्य व संक्रामक है, परन्तु सममित नहीं है।

अथवा

सिद्ध कीजिए कि फलन $f : \mathbb{N} \rightarrow \mathbb{N}$, $f(x) = x^2 + x + 1$, द्वारा परिभाषित है, एक एकैकी फलन है किंतु आच्छादक नहीं।

फलन $f : \mathbb{N} \rightarrow S$, जहाँ S फलन f का परिसर है, का प्रतिलोम भी ज्ञात कीजिए।

Show that the relation R on \mathbb{R} defined as $R = \{(a, b) : a \leq b\}$, is reflexive, and transitive but not symmetric.

OR

Prove that the function $f : \mathbb{N} \rightarrow \mathbb{N}$, defined by $f(x) = x^2 + x + 1$ is one-one but not onto. Find inverse of $f : \mathbb{N} \rightarrow S$, where S is range of f .

14. हल कीजिए : $\tan^{-1} 4x + \tan^{-1} 6x = \frac{\pi}{4}$

Solve : $\tan^{-1} 4x + \tan^{-1} 6x = \frac{\pi}{4}$.

15. सारणिकों के गुणधर्मों का प्रयोग करके, सिद्ध कीजिए कि $\begin{vmatrix} a^2 + 2a & 2a + 1 & 1 \\ 2a + 1 & a + 2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (a - 1)^3$.

Using properties of determinants, prove that $\begin{vmatrix} a^2 + 2a & 2a + 1 & 1 \\ 2a + 1 & a + 2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (a - 1)^3$.

16. यदि $\log(x^2 + y^2) = 2 \tan^{-1} \left(\frac{y}{x}\right)$ हो, तो दर्शाइए कि $\frac{dy}{dx} = \frac{x + y}{x - y}$.

अथवा

यदि $x^y - y^x = a^b$ है, तो $\frac{dy}{dx}$ ज्ञात कीजिए।

If $\log(x^2 + y^2) = 2 \tan^{-1} \left(\frac{y}{x}\right)$, show that $\frac{dy}{dx} = \frac{x + y}{x - y}$.

OR

If $x^y - y^x = a^b$, find $\frac{dy}{dx}$.



17. यदि $y = (\sin^{-1}x)^2$ है, तो सिद्ध कीजिए कि $(1 - x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} - 2 = 0$.

If $y = (\sin^{-1}x)^2$, prove that $(1 - x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} - 2 = 0$.

18. वक्र $y = \sqrt{3x - 2}$ की उस स्पर्श-रेखा का समीकरण ज्ञात कीजिए जो रेखा $4x - 2y + 5 = 0$ के समान्तर है। स्पर्श बिन्दु से वक्र पर बने अभिलंब का समीकरण भी ज्ञात कीजिए।

Find the equation of tangent to the curve $y = \sqrt{3x - 2}$ which is parallel to the line $4x - 2y + 5 = 0$. Also, write the equation of normal to the curve at the point of contact.

19. ज्ञात कीजिए : $\int \frac{3x + 5}{x^2 + 3x - 18} dx$.

Find : $\int \frac{3x + 5}{x^2 + 3x - 18} dx$.

20. सिद्ध कीजिए कि $\int_0^a f(x) dx = \int_0^a f(a - x) dx$, अतः $\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx$ का मान ज्ञात कीजिए।

Prove that $\int_0^a f(x) dx = \int_0^a f(a - x) dx$, hence evaluate $\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx$.



21. अवकल समीकरण : $x dy - y dx = \sqrt{x^2 + y^2} dx$ को हल कीजिए, दिया गया है $y = 0$ यदि $x = 1$.

अथवा

अवकल समीकरण : $(1 + x^2) \frac{dy}{dx} + 2xy - 4x^2 = 0$ को हल कीजिए, दिया गया है $y(0) = 0$.

Solve the differential equation : $x dy - y dx = \sqrt{x^2 + y^2} dx$, given that $y = 0$ when $x = 1$.

OR

Solve the differential equation : $(1 + x^2) \frac{dy}{dx} + 2xy - 4x^2 = 0$, subject to the initial condition $y(0) = 0$.

22. यदि $\hat{i} + \hat{j} + \hat{k}$, $2\hat{i} + 5\hat{j}$, $3\hat{i} + 2\hat{j} - 3\hat{k}$ तथा $\hat{i} - 6\hat{j} - \hat{k}$ क्रमशः बिन्दु A, B, C और D के स्थिति सदिश हों तो सरल रेखाओं AB तथा CD के बीच का कोण ज्ञात कीजिए। ज्ञात कीजिए कि क्या सदिश \vec{AB} तथा \vec{CD} संरेख हैं या नहीं।

If $\hat{i} + \hat{j} + \hat{k}$, $2\hat{i} + 5\hat{j}$, $3\hat{i} + 2\hat{j} - 3\hat{k}$ and $\hat{i} - 6\hat{j} - \hat{k}$ respectively are the position vectors of points A, B, C and D, then find the angle between the straight lines AB and CD. Find whether \vec{AB} and \vec{CD} are collinear or not.

23. λ का वह मान ज्ञात कीजिए जिसके लिए निम्न रेखाएँ लम्बवत हैं :

$\frac{1-x}{3} = \frac{7y-14}{\lambda} = \frac{z-3}{2}$ तथा $\frac{7-7x}{3\lambda} = \frac{y-5}{1} = \frac{6-z}{5}$, यह भी ज्ञात कीजिए कि क्या ये रेखाएँ परस्पर प्रतिच्छेद करती हैं या नहीं।

Find the value of λ , so that the lines $\frac{1-x}{3} = \frac{7y-14}{\lambda} = \frac{z-3}{2}$ and $\frac{7-7x}{3\lambda} = \frac{y-5}{1} = \frac{6-z}{5}$ are at right angles. Also, find whether the lines are intersecting or not.



खण्ड – द

SECTION – D

प्रश्न संख्या 24 से 29 के प्रत्येक प्रश्न के 6 अंक हैं।

Question numbers 24 to 29 carry 6 marks each.

24. यदि $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix}$ हो, तो A^{-1} ज्ञात कीजिए।

अतः निम्न समीकरण निकाय को हल कीजिए :

$$x + y + z = 6$$

$$x + 2z = 7$$

$$3x + y + z = 12$$

अथवा

प्रारंभिक संक्रियाओं द्वारा निम्न आव्यूह का प्रतिलोम ज्ञात कीजिए :

$$A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix}$, find A^{-1} . Hence, solve the system of equations

$$x + y + z = 6, x + 2z = 7, 3x + y + z = 12.$$

OR

Find the inverse of the following matrix using elementary operations.

$$A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

25. आयताकार आधार व आयताकार दीवारों की 2 m गहरी और 8 m^3 आयतन की एक बिना ढक्कन की टंकी का निर्माण करना है। यदि टंकी के निर्माण में आधार के लिए ₹ $70/\text{m}^2$ और दीवारों पर ₹ $45/\text{m}^2$ व्यय आता है तो न्यूनतम खर्च से बनी टंकी की लागत क्या है ?

A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 2 m and volume is 8 m^3 . If building of tank costs ₹ 70 per square metre for the base and ₹ 45 per square metre for the sides, what is the cost of least expensive tank ?



26. समाकलन विधि द्वारा, त्रिभुज ABC का क्षेत्रफल ज्ञात कीजिए, जहाँ A(2, 5), B (4, 7) तथा C(6, 2) त्रिभुज ABC के शीर्ष हैं।

अथवा

समाकलन विधि से, x -अक्ष से ऊपर तथा वृत्त $x^2 + y^2 = 8x$ एवं परवलय $y^2 = 4x$ के अंतः भाग के मध्यवर्ती क्षेत्र का क्षेत्रफल ज्ञात कीजिए।

Using integration, find the area of triangle ABC, whose vertices are A(2, 5), B(4, 7) and C(6, 2).

OR

Find the area of the region lying above x -axis and included between the circle $x^2 + y^2 = 8x$ and inside of the parabola $y^2 = 4x$.

27. बिन्दुओं (2, 2, -1), (3, 4, 2) तथा (7, 0, 6) से गुजरने वाले समतल के सदिश व कार्तीय समीकरण ज्ञात कीजिए। अतः उस समतल का समीकरण ज्ञात कीजिए जो बिन्दु (4, 3, 1) से गुजरता है और ऊपर प्राप्त समतल के समान्तर है।

अथवा

उस समतल का सदिश समीकरण ज्ञात कीजिए जो रेखा $\vec{r} = (\hat{i} + \hat{j}) + \lambda (\hat{i} + 2\hat{j} - \hat{k})$ तथा बिन्दु (-1, 3, -4) को अंतर्विष्ट करता है। इस समतल पर बिन्दु (2, 1, 4) से डाले गए लंब की दूरी भी ज्ञात कीजिए।

Find the vector and Cartesian equations of the plane passing through the points (2, 2 -1), (3, 4, 2) and (7, 0, 6). Also find the vector equation of a plane passing through (4, 3, 1) and parallel to the plane obtained above.

OR

Find the vector equation of the plane that contains the lines $\vec{r} = (\hat{i} + \hat{j}) + \lambda (\hat{i} + 2\hat{j} - \hat{k})$ and the point (-1, 3, -4). Also, find the length of the perpendicular drawn from the point (2, 1, 4) to the plane, thus obtained.



28. एक निर्माता के पास तीन कारीगर A, B तथा C हैं। कारीगर A 1% त्रुटिपूर्ण इकाइयों का उत्पादन करता है, कारीगर B 5% तथा कारीगर C 7% त्रुटिपूर्ण इकाइयों का उत्पादन करते हैं। A सिर्फ 50% समय ही उत्पादन करता है, B सिर्फ 30% समय व C सिर्फ 20% समय ही उत्पादन करते हैं। यदि कुल उत्पादन का एक ढेर बना लिया जाता है और उस ढेर से यादृच्छया निकाली गई एक इकाई त्रुटिपूर्ण हो, तो इस इकाई के A द्वारा बनाई गई होने की प्रायिकता ज्ञात कीजिए।

A manufacturer has three machine operators A, B and C. The first operator A produces 1% of defective items, whereas the other two operators B and C produces 5% and 7% defective items respectively. A is on the job for 50% of the time, B on the job 30% of the time and C on the job for 20% of the time. All the items are put into one stockpile and then one item is chosen at random from this and is found to be defective. What is the probability that it was produced by A ?

29. एक निर्माता 5 कुशल व 10 अर्धकुशल कारीगरों को काम पर रखता है और एक उत्पाद के दो नमूने A और B बनाता है। नमूना A के प्रत्येक नग बनाने के लिए कुशल कारीगर को 2 घंटे व अर्धकुशल कारीगर को 2 घंटे काम करना पड़ता है। नमूना B के प्रत्येक नग बनाने के लिए कुशल कारीगर को 1 घंटा व अर्ध-कुशल कारीगर को 3 घंटे काम करना पड़ता है। दोनों ही कारीगरों में प्रत्येक को काम करने के लिए प्रतिदिन अधिकतम 8 घण्टे का समय उपलब्ध है। निर्माता को नमूने A के प्रत्येक नग पर ₹ 15 लाभ व नमूने B के प्रत्येक नग पर ₹ 10 का लाभ होता है। नमूना A और नमूना B के कितने नगों का अधिकतम लाभ कमाने के लिए, प्रतिदिन निर्माण करना चाहिए ? इस प्रश्न को रैखिक प्रोग्रामन समस्या के रूप में लिखिए और ग्राफ द्वारा हल कीजिए। अधिकतम लाभ भी ज्ञात कीजिए।

A manufacturer has employed 5 skilled men and 10 semi-skilled men and makes two models A and B of an article. The making of one item of model A requires 2 hours work by a skilled man and 2 hours work by a semi-skilled man. One item of model B requires 1 hour by a skilled man and 3 hours by a semi-skilled man. No man is expected to work more than 8 hours per day. The manufacturer's profit on an item of model A is ₹ 15 and on an item of model B is ₹ 10. How many of items of each model should be made per day in order to maximize daily profit ? Formulate the above LPP and solve it graphically and find the maximum profit.



रोल नं.

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Roll No.

परीक्षार्थी कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें।

Candidates must write the Code on the title page of the answer-book.

- कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ 8 हैं।
- प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए कोड नम्बर को छात्र उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें।
- कृपया जाँच कर लें कि इस प्रश्न-पत्र में 29 प्रश्न हैं।
- कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, प्रश्न का क्रमांक अवश्य लिखें।
- इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है। प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा। 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे।
- Please check that this question paper contains 8 printed pages.
- Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains 29 questions.
- **Please write down the Serial Number of the question before attempting it.**
- 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

गणित

MATHEMATICS

निर्धारित समय : 3 घण्टे

Time allowed : 3 hours

अधिकतम अंक : 100

Maximum Marks : 100

सामान्य निर्देश :

- सभी प्रश्न अनिवार्य हैं।
- इस प्रश्न-पत्र में 29 प्रश्न हैं जो चार खण्डों में विभाजित हैं : अ, ब, स तथा द। खण्ड अ में 4 प्रश्न हैं जिनमें से प्रत्येक एक अंक का है। खण्ड ब में 8 प्रश्न हैं जिनमें से प्रत्येक दो अंक का है। खण्ड स में 11 प्रश्न हैं जिनमें से प्रत्येक चार अंक का है। खण्ड द में 6 प्रश्न हैं जिनमें से प्रत्येक छः अंक का है।
- खण्ड अ में सभी प्रश्नों के उत्तर एक शब्द, एक वाक्य अथवा प्रश्न की आवश्यकतानुसार दिए जा सकते हैं।
- पूर्ण प्रश्न-पत्र में विकल्प नहीं हैं। फिर भी चार अंकों वाले 3 प्रश्नों में तथा छः अंकों वाले 3 प्रश्नों में आंतरिक विकल्प है। ऐसे सभी प्रश्नों में से आपको एक ही विकल्प हल करना है।
- कैलकुलेटर के प्रयोग की अनुमति नहीं है। यदि आवश्यक हो, तो आप लघुगणकीय सारणियाँ माँग सकते हैं।

General Instructions :

- (i) *All questions are compulsory.*
- (ii) *The question paper consists of 29 questions divided into four sections A, B, C and D. Section A comprises of 4 questions of **one mark** each, Section B comprises of 8 questions of **two marks** each, Section C comprises of 11 questions of **four marks** each and Section D comprises of 6 questions of **six marks** each.*
- (iii) *All questions in Section A are to be answered in **one** word, **one** sentence or as per the exact requirement of the question.*
- (iv) *There is no overall choice. However, internal choice has been provided in 3 questions of **four** marks each and 3 questions of **six** marks each. You have to attempt only **one** of the alternatives in all such questions.*
- (v) *Use of calculators is **not** permitted. You may ask for logarithmic tables, if required.*

खण्ड – अ

SECTION – A

प्रश्न संख्या 1 से 4 तक प्रत्येक प्रश्न 1 अंक का है ।

Question numbers 1 to 4 carry 1 mark each.

1. $\tan^{-1} \sqrt{3} - \sec^{-1} (-2)$ का मान ज्ञात कीजिए ।

Find the value of $\tan^{-1} \sqrt{3} - \sec^{-1} (-2)$.

2. यदि $A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & x \\ -2 & 2 & -1 \end{pmatrix}$ ऐसा आव्यूह है जो $AA' = 9I$ को संतुष्ट करता है, तो x ज्ञात कीजिए ।

If $A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & x \\ -2 & 2 & -1 \end{pmatrix}$ is a matrix satisfying $AA' = 9I$, find x .

3. $[\hat{i}, \hat{k}, \hat{j}]$ का मान ज्ञात कीजिए ।

Find the value of $[\hat{i}, \hat{k}, \hat{j}]$.

4. समुच्चय Q^+ जो सभी धन परिमेय संख्याओं का समुच्चय है, में संक्रिया $*$, जो सभी $a, b \in Q_+$ के लिए

$a * b = \frac{3ab}{2}$ द्वारा परिभाषित है, का तत्समक अवयव ज्ञात कीजिए ।

Find the identity element in the set Q^+ of all positive rational numbers for the operation $*$ defined by $a * b = \frac{3ab}{2}$ for all $a, b \in Q_+$.

खण्ड – ब
SECTION – B

प्रश्न संख्या 5 से 12 तक प्रत्येक प्रश्न के 2 अंक हैं ।

Question numbers 5 to 12 carry 2 marks each.

5. सिद्ध कीजिए कि $3 \cos^{-1} x = \cos^{-1} (4x^3 - 3x)$, $x \in \left[\frac{1}{2}, 1 \right]$.

Prove that $3 \cos^{-1} x = \cos^{-1} (4x^3 - 3x)$, $x \in \left[\frac{1}{2}, 1 \right]$.

6. यदि $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ ऐसा है कि $A^{-1} = kA$ है, तो k का मान ज्ञात कीजिए ।

If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that $A^{-1} = kA$, then find the value of k .

7. $\tan^{-1} \left(\frac{\cos x - \sin x}{\cos x + \sin x} \right)$ का x के सापेक्ष अवकलन कीजिए ।

Differentiate $\tan^{-1} \left(\frac{\cos x - \sin x}{\cos x + \sin x} \right)$ with respect to x .

8. किसी उत्पाद की x -इकाइयों के विक्रय से प्राप्त कुल आय ₹ में $R(x) = 3x^2 + 36x + 5$ से प्रदत्त है। जब $x = 5$ है, तो सीमांत आय ज्ञात कीजिए, जहाँ सीमांत आय से अभिप्राय किसी क्षण विक्रय की गई वस्तुओं के संपूर्ण आय के परिवर्तन की दर से है ।

The total revenue received from the sale of x units of a product is given by $R(x) = 3x^2 + 36x + 5$ in rupees. Find the marginal revenue when $x = 5$, where by marginal revenue we mean the rate of change of total revenue with respect to the number of items sold at an instant.

9. ज्ञात कीजिए : $\int \frac{3 - 5 \sin x}{\cos^2 x} dx$

Find : $\int \frac{3 - 5 \sin x}{\cos^2 x} dx$.

10. अवकल समीकरण $\cos \left(\frac{dy}{dx} \right) = a$, ($a \in \mathbb{R}$) को हल कीजिए ।

Solve the differential equation $\cos \left(\frac{dy}{dx} \right) = a$, ($a \in \mathbb{R}$).

11. यदि $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ तथा $|\vec{a}| = 5$, $|\vec{b}| = 6$ तथा $|\vec{c}| = 9$ है, तो \vec{a} तथा \vec{b} के बीच का कोण ज्ञात कीजिए।

If $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ and $|\vec{a}| = 5$, $|\vec{b}| = 6$ and $|\vec{c}| = 9$, then find the angle between \vec{a} and \vec{b} .

12. यदि $2P(A) = P(B) = \frac{5}{13}$ तथा $P(A/B) = \frac{2}{5}$ है, तो $P(A \cup B)$ का मान ज्ञात कीजिए।

Evaluate $P(A \cup B)$, if $2P(A) = P(B) = \frac{5}{13}$ and $P(A/B) = \frac{2}{5}$.

खण्ड – स

SECTION – C

प्रश्न संख्या 13 से 23 तक प्रत्येक प्रश्न के 4 अंक हैं।

Question numbers 13 to 23 carry 4 marks each.

13. सारणिकों के गुणधर्मों का प्रयोग कर, सिद्ध कीजिए कि

$$\begin{vmatrix} 5a & -2a+b & -2a+c \\ -2b+a & 5b & -2b+c \\ -2c+a & -2c+b & 5c \end{vmatrix} = 12(a+b+c)(ab+bc+ca)$$

Using properties of determinants, prove that

$$\begin{vmatrix} 5a & -2a+b & -2a+c \\ -2b+a & 5b & -2b+c \\ -2c+a & -2c+b & 5c \end{vmatrix} = 12(a+b+c)(ab+bc+ca)$$

14. यदि $\sin y = x \cos(a+y)$ है, तो दर्शाइए कि $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\cos a}$.

यह भी दर्शाइए कि $\frac{dy}{dx} = \cos a$ है, जब $x = 0$ है।

If $\sin y = x \cos(a+y)$, then show that $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\cos a}$.

Also, show that $\frac{dy}{dx} = \cos a$, when $x = 0$.

15. यदि $x = a \sec^3 \theta$ तथा $y = a \tan^3 \theta$ है, तो $\theta = \frac{\pi}{3}$ पर $\frac{d^2y}{dx^2}$ ज्ञात कीजिए ।

अथवा

यदि $y = e^{\tan^{-1} x}$ है, तो सिद्ध कीजिए कि $(1 + x^2) \frac{d^2y}{dx^2} + (2x - 1) \frac{dy}{dx} = 0$

If $x = a \sec^3 \theta$ and $y = a \tan^3 \theta$, find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{3}$.

OR

If $y = e^{\tan^{-1} x}$, prove that $(1 + x^2) \frac{d^2y}{dx^2} + (2x - 1) \frac{dy}{dx} = 0$.

16. वक्र $x^2 + y^2 = 4$ तथा $(x - 2)^2 + y^2 = 4$ प्रथम चतुर्थांश में किसी बिंदु पर किस कोण पर काटते हैं ?

अथवा

वह अंतराल ज्ञात कीजिए जिनमें फलन $f(x) = -2x^3 - 9x^2 - 12x + 1$

(i) निरंतर वर्धमान है । (ii) निरंतर ह्रासमान है ।

Find the angle of intersection of the curves $x^2 + y^2 = 4$ and $(x - 2)^2 + y^2 = 4$, at the point in the first quadrant.

OR

Find the intervals in which the function $f(x) = -2x^3 - 9x^2 - 12x + 1$ is

(i) Strictly increasing (ii) Strictly decreasing

17. किसी आयत के ऊपर बने अर्धवृत्त के आकार की एक खिड़की है । खिड़की का संपूर्ण परिमाण 10 मीटर है । पूर्णतया खुली खिड़की से अधिकतम प्रकाश आने के लिए खिड़की की विमाएँ ज्ञात कीजिए । बड़ी खिड़कियाँ होने पर कैसे बिजली की बचत होती है तथा वातावरण का संतुलन बना रहता है ?

A window is in the form of a rectangle surmounted by a semicircular opening. The total perimeter of the window is 10 metres. Find the dimensions of the window to admit maximum light through the whole opening. How having large windows help us in saving electricity and conserving environment ?

18. ज्ञात कीजिए : $\int \frac{4}{(x-2)(x^2+4)} dx$

Find : $\int \frac{4}{(x-2)(x^2+4)} dx$

19. अवकल समीकरण $(x^2 - y^2) dx + 2xydy = 0$ का हल कीजिए ।

अथवा

अवकल समीकरण $(1 + x^2) \frac{dy}{dx} + 2xy = \frac{1}{1+x^2}$ का विशिष्ट हल ज्ञात कीजिए, दिया है जब $x = 1$ है

तो $y = 0$ है ।

Solve the differential equation $(x^2 - y^2) dx + 2xydy = 0$

OR

Find the particular solution of the differential equation $(1 + x^2) \frac{dy}{dx} + 2xy = \frac{1}{1+x^2}$,

given that $y = 0$ when $x = 1$.

20. x का मान ज्ञात कीजिए कि चार बिंदु $A(4, 4, 4)$, $B(5, x, 8)$, $C(5, 4, 1)$ तथा $D(7, 7, 2)$ समतलीय हों ।

Find x such that the four points $A(4, 4, 4)$, $B(5, x, 8)$, $C(5, 4, 1)$ and $D(7, 7, 2)$ are coplanar.

21. रेखाओं $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ तथा $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ के बीच न्यूनतम दूरी ज्ञात कीजिए ।

Find the shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$.

22. दो दल एक निगम के निदेशक मंडल में स्थान पाने की प्रतिस्पर्धा में हैं । पहले तथा दूसरे दल के जीतने की प्रायिकताएँ क्रमशः 0.6 तथा 0.4 हैं । इसके अतिरिक्त यदि पहला दल जीतता है तो एक नए उत्पाद के आरंभ होने की प्रायिकता 0.7 है और यदि दूसरा दल जीतता है तो इस बात की संगत प्रायिकता 0.3 है । प्रायिकता ज्ञात कीजिए कि नया उत्पाद दूसरे दल द्वारा आरंभ किया गया था ।

Two groups are competing for the positions of the Board of Directors of a corporation. The probabilities that the first and second groups will win are 0.6 and 0.4 respectively. Further, if the first group wins, the probability of introducing a new product is 0.7 and the corresponding probability is 0.3 if the second group wins. Find the probability that the new product introduced was by the second group.

23. 20 बल्बों के एक ढेर से, जिसमें 5 बल्ब खराब हैं, 3 बल्बों का एक नमूना यादृच्छया एक-एक करके प्रतिस्थापना सहित निकाला गया । खराब बल्बों की संख्या का प्रायिकता बटन ज्ञात कीजिए । अतः इस बटन की माध्य भी ज्ञात कीजिए ।

From a lot of 20 bulbs which include 5 defectives, a sample of 3 bulbs is drawn at random, one by one with replacement. Find the probability distribution of the number of defective bulbs. Also, find the mean of the distribution.

खण्ड – द
SECTION – D

प्रश्न संख्या 24 से 29 तक प्रत्येक प्रश्न के 6 अंक हैं ।

Question numbers 24 to 29 carry 6 marks each.

24. दर्शाइए कि सभी पूर्णाकों के समुच्चय Z में एक संबंध R , जो कि $(x, y) \in R \Leftrightarrow (x - y)$, 3 से भाज्य है, द्वारा परिभाषित है, एक तुल्यता संबंध है ।

अथवा

समुच्चय $A = \{0, 1, 2, 3, 4, 5\}$ पर एक द्विआधारी संक्रिया $*$ जो $a * b = \begin{cases} a + b, & \text{यदि } a + b < 6 \text{ है} \\ a + b - 6, & \text{यदि } a + b \geq 6 \text{ है} \end{cases}$ द्वारा परिभाषित है ।

A में $a * b$ के लिए संक्रिया सारणी लिखिए ।

दर्शाइए कि संक्रिया $*$ के लिए 0 एक तत्समक अवयव है तथा समुच्चय A का प्रत्येक अवयव $a \neq 0$ व्युत्क्रमणीय है, इस प्रकार कि $6 - a$, a का प्रतिलोम है ।

Show that the relation R on the set Z of all integers defined by $(x, y) \in R \Leftrightarrow (x - y)$ is divisible by 3 is an equivalence relation.

OR

A binary operation $*$ on the set $A = \{0, 1, 2, 3, 4, 5\}$ is defined as

$$a * b = \begin{cases} a + b, & \text{if } a + b < 6 \\ a + b - 6, & \text{if } a + b \geq 6 \end{cases}$$

Write the operation table for $a * b$ in A .

Show that zero is the identity for this operation $*$ and each element ' a ' $\neq 0$ of the set is invertible with $6 - a$, being the inverse of ' a '.

25. दिया है कि $A = \begin{bmatrix} 5 & 0 & 4 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{bmatrix}$, $B^{-1} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ है, तो $(AB)^{-1}$ ज्ञात कीजिए ।

अथवा

प्रारंभिक पंक्ति रूपांतरणों द्वारा आव्यूह $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ का व्युत्क्रम ज्ञात कीजिए ।

Given $A = \begin{bmatrix} 5 & 0 & 4 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{bmatrix}$, $B^{-1} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$, compute $(AB)^{-1}$.

OR

Find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ by using elementary row transformations.

26. समाकलनों के प्रयोग से निम्न क्षेत्र का क्षेत्रफल ज्ञात कीजिए : $\{(x, y) : 0 \leq 2y \leq x^2, 0 \leq y \leq x, 0 \leq x \leq 3\}$
Using integration, find the area of the region : $\{(x, y) : 0 \leq 2y \leq x^2, 0 \leq y \leq x, 0 \leq x \leq 3\}$

27. मान ज्ञात कीजिए : $\int_0^{\frac{\pi}{2}} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx$

अथवा

योगों की सीमा के रूप में $\int_1^3 (3x^2 + 2x + 1) dx$ का मान ज्ञात कीजिए ।

Evaluate $\int_0^{\frac{\pi}{2}} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx$.

OR

Evaluate $\int_1^3 (3x^2 + 2x + 1) dx$ as the limit of a sum.

28. उस रेखा का सदिश समीकरण ज्ञात कीजिए जो बिंदु (1, 2, 3) से होकर जाती है तथा समतलों $\vec{r} \cdot (\hat{i} - \hat{j} + 2\hat{k}) = 5$ तथा $\vec{r} \cdot (3\hat{i} + \hat{j} + \hat{k}) = 6$ में प्रत्येक के समांतर है । इस प्रकार प्राप्त रेखा का समतल $\vec{r} \cdot (2\hat{i} + \hat{j} + \hat{k}) = 4$ से प्रतिच्छेदन बिंदु ज्ञात कीजिए ।

Find the vector equation of the line passing through (1, 2, 3) and parallel to each of the planes $\vec{r} \cdot (\hat{i} - \hat{j} + 2\hat{k}) = 5$ and $\vec{r} \cdot (3\hat{i} + \hat{j} + \hat{k}) = 6$. Also find the point of intersection of the line thus obtained with the plane $\vec{r} \cdot (2\hat{i} + \hat{j} + \hat{k}) = 4$.

29. एक कंपनी दो प्रकार की वस्तुओं A तथा B का निर्माण करती है, जिनमें सोने तथा चाँदी का प्रयोग होता है । A प्रकार की वस्तु की एक इकाई में 3 ग्राम चाँदी तथा 1 ग्राम सोने का प्रयोग होता है जबकि वस्तु B की एक इकाई के लिए 1 ग्राम चाँदी तथा 2 ग्राम सोने का प्रयोग होता है । कंपनी अधिक से अधिक 9 ग्राम चाँदी तथा 8 ग्राम सोना प्रयोग कर सकती है । यदि A प्रकार की वस्तु की एक इकाई पर ₹ 40 का लाभ मिलता है तथा वस्तु B की एक इकाई पर ₹ 50 का लाभ मिलता है, तो ज्ञात कीजिए कि कंपनी A तथा B प्रकार की वस्तुएँ कितनी-कितनी बनाएँ कि कंपनी को अधिकतम लाभ हो । उपरोक्त प्रश्न को एक रैखिक प्रोग्रामन समस्या बनाकर ग्राफ द्वारा हल कीजिए तथा अधिकतम लाभ भी ज्ञात कीजिए ।

A company produces two types of goods, A and B, that require gold and silver. Each unit of type A requires 3 g of silver and 1 g of gold while that of B requires 1 g of silver and 2 g of gold. The company can use atmost 9 g of silver and 8 g of gold. If each unit of type A brings a profit of ₹ 40 and that of type B ₹ 50, find the number of units of each type that the company should produce to maximize the profit. Formulate and solve graphically the LPP and find the maximum profit.

PRACTICE PAPER

CLASS XII

MATHEMATICS

TIME : 3 HOURS

Max Marks : 80

General Instruction:

1. All questions are compulsory
2. This question paper contains 36 questions.
3. Section-A comprises of twenty questions (Question 1-20) of 1 mark each,
4. Section-B comprises of six questions (Question 21-26) of 2 marks each,
5. Section-C comprises of six questions (Questions 27-32) of 4 marks each
6. Section-D comprises of four questions (Question 33-36) of 6 marks each.
7. There is no overall choice but internal choices are given in section B,C and D.
8. Use of calculator is prohibited.

SECTION A

1. If for the matrix A, $|A| = 5$, find $|4A|$, where matrix A is of order 2×2
(a) 20 (b) 80 (c) 4 (d) 16
2. If f is an invertible function, find the inverse of $f(x) = \frac{3x-2}{5}$.
3. The value of $\cos \left[\frac{\pi}{3} - \sin^{-1} \left(\frac{-1}{2} \right) \right]$ is
(a) $\frac{-\sqrt{3}}{2}$ (b) 0 (c) $\frac{1}{2}$ (d) -1
4. A singular matrix is invertible, state true or false
5. Derivative of $\sqrt{e^{\sqrt{x}}}$ with respect to x
(a) $\sqrt{e^{\sqrt{x}}}$ (b) $e^{\sqrt{x}}$
(c) $\frac{e^{\sqrt{x}}}{2\sqrt{x}\sqrt{e^{\sqrt{x}}}}$ (d) $\frac{\sqrt{e^{\sqrt{x}}}}{4\sqrt{x}}$
6. For what value of K the function $f(x) = \frac{x^2-1}{x-1}$, if $x \neq 1$
 $4k$, if $x=1$
Is continuous at $x=1$
(a) 2 (b) 1 (c) 4 (d) $\frac{1}{2}$
7. Derivative of x^x with respect to x is
(a) $(1+\log x)$ (b) $x \cdot x^{x-1}$ (c) $x \log x$ (d) $x^x (1+\log x)$
8. Rate of change of volume of a sphere of diameter r , with respect to r is
(a) $4\pi r^2$ (b) $\frac{4}{3}\pi r^2$ (c) $\frac{1}{2}\pi r^2$ (d) πr^2

- 9 Find the slope of normal to the curve , $x = a\cos^3\theta$, $y = a\sin^3\theta$, $\theta = \frac{\pi}{4}$
- 10 Without using derivative find the maximum and minimum values of $|3 \sin x + 2|$, $x \in R$
- 11 The value of $\int_0^1 \frac{1}{1+x^2} dx$ is
 (a) $\log 2$ (b) $\frac{1}{\log 2}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{4}$
- 12 If $\int e^{-2 \log x} dx = f(x) + C$, then $f(x)$ is
 (a) $\frac{e^{-2 \cos x + 1}}{-2 \log x + 1}$ (b) $-2 \log x \cdot e^{-2 \log x}$ (c) $\frac{1}{x^2}$ (d) $-\frac{1}{x}$
- 13 Evaluate : $\int \frac{dx}{\sin^2 x \cos^2 x}$
- 14 Degree of differential equation $\frac{d^2y}{dx^2} + \sin\left(\frac{dy}{dx}\right) = 0$ is 1. State true or false
- 15 Integrating factor for differential equation $\frac{dy}{dx} + y \tan x - \sec x = 0$ is
- 16 Direction cosines of y-axis are
 (a) (1,0,0) (b) 0,1,0 (c) (0,1,0) (d) (0,0,1)
- 17 If $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} = \vec{0}$ Then vectors $\vec{a}, \vec{b}, \vec{c}$ are
 (a) parallel (b) collinear (c) coplanar (d) non-coplanar
- 18 A die is thrown once if odd number turns up, what is the probability that die shows a prime number ?
- 19 Let E and F be the events with $P(E) = \frac{3}{5}$ $P(F) = \frac{3}{10}$ $P(E \cap F) = \frac{1}{5}$, then the events E and F are not independent. State true or false.
- 20 The maximum value of objective function $z = ax + by$ in LPP always occurs at only one corner point of the feasible region. State true or false.

SECTION B

- 21 Evaluate $\sin\left(\frac{1}{2} \cos^{-1} \frac{4}{5}\right)$
- 22 If A and B are symmetric matrices. Prove that $AB - BA$ is a skew-symmetric matrix
- 23 For what values of x the matrix $\begin{bmatrix} x & 1 & 2 \\ 1 & 0 & 3 \\ 5 & -1 & 4 \end{bmatrix}$ is singular.

OR

If $A^{-1} = \begin{bmatrix} 0 & 1 \\ 4 & 3 \end{bmatrix}$, $B^{-1} = \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix}$, Find $(AB)^{-1}$

- 24 Find the point(s) on the curve $2y = 3 - x^2$, at which the tangent is parallel to the line $x + y = 0$.

OR

Show that the tangents to the curve $y = 7x^3 + 11$, are parallel at the points where $x = 2$, $x = -2$.

- 25 If $y = \cos^{-1} \left(\frac{x^2 - 1}{x^2 + 1} \right)$, find $\frac{dy}{dx}$.

SECTION C

- 26 If $P(\text{not } A) = 0.7$, $P(B) = 0.7$, and $P(B/A) = 0.5$, find $P(A/B)$.

- 27 Using properties of determinants, solve for x

$$\begin{vmatrix} x - 2 & 2x - 3 & 3x - 4 \\ x - 4 & 2x - 9 & 3x - 16 \\ x - 8 & 2x - 27 & 3x - 64 \end{vmatrix} = 0$$

OR Show that

$$\begin{vmatrix} a^2 + 1 & ab & ac \\ ab & b^2 + 1 & bc \\ ca & cb & c^2 + 1 \end{vmatrix} = 1 + a^2 + b^2 + c^2$$

- 28 N is a set of natural numbers, show that the relation r in $N \times N$ defined by $(a, b) R (c, d) \Leftrightarrow a + d = b + c$, for all $(a, b), (c, d) \in N \times N$ is an equivalence relation.

- 29 Evaluate $\int \frac{\log x}{(x+1)^2} dx$

OR

$\int_1^3 (x^2 + x) dx$ as limit of sum.

- 30 An insurance company insured 2000 scooter drivers, 3000 cyclists and 4000 motor bikers. The Probabilities of their meeting with an accident respectively are 0.04, 0.06 and 0.15. One of the insured persons meets with an accident, find the probability that he is a scooter driver.

- 31 Find the shortest distance between the lines

$$\vec{r} = (1 + \lambda)\vec{i} + (2 - \lambda)\vec{j} + (\lambda + 1)\vec{k};$$

$$\vec{r} = (2\vec{i} - \vec{j} - \vec{k}) + \mu (2\vec{i} + \vec{j} + 2\vec{k})$$

OR

Find μ , so that the four points with position vectors $-6\hat{i}+3\hat{j}+2\hat{k}$, $3\hat{i}+\mu\hat{j}+4\hat{k}$, $5\hat{i}+7\hat{j}+3\hat{k}$ and $-13\hat{i}+17\hat{j}-\hat{k}$ are coplanar.

- 32 An aeroplane can carry maximum of 200 passengers. A profit of Rs.1,000 is made on each economy class ticket and a profit of Rs.600 is made on each executive class ticket. The airline reserves at least 20 seats for the executive class. However, at least 4 times as many passengers prefer to travel by economy class than by executive class. Determine how many tickets of each type must be sold, in order to maximize the profit for the airline? What is the maximum profit? Make an LPP and solve it graphically.

SECTION-D

- 33 Find the area of the greatest rectangle that can be inscribed in an ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

- 34 Using integration, find the area of the region bounded by $\{(x, y): x^2 + y^2 \leq 1 \leq x + y\}$

OR

Using integration, Find the area of circle $x^2 + y^2 = 16$, which is exterior to the parabola, $y^2 = 6x$

- 35 Solve the differential equation $(1 + y^2)dx = (\tan^{-1} y - x) dy$

OR

Show that the differential equation $\left[x \sin^2\left(\frac{y}{x}\right) - y\right] dx + x dy = 0$ is homogeneous. Find the particular solution given that $y = \frac{\pi}{4}$, when $x = 1$.

- 36 Find the equation of the perpendicular drawn from the point $P(2,4,-1)$ to the line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$. Also write down the coordinates of the foot of the perpendicular from point P to the line.

ANSWERS :

1. b 2. $\frac{5x+2}{3}$ 3.b 4. False 5. d 6. d 7. d 8.c
9. 1 , 10. Minimum value= 0, maximum value = 5
- 11.c 12. d 13.tanx – cotx +C 14. False 15.sec x 16.b 17.c 18.2/3 19. True
- 20 False
21. $\frac{1}{\sqrt{10}}$
- 23.-3, $\begin{bmatrix} -4 & -2 \\ 16 & 14 \end{bmatrix}$ 24. (1,1)
25. $\frac{-2}{1+x^2}$ 26. $\frac{3}{14}$
27. x = 4 29. $\frac{-1}{x+1} \log x + \log x - \log|x + 1| + C , \frac{38}{3}$
30. $\frac{4}{43}$, 31. $\frac{3\sqrt{2}}{2}$, -2
32. 40executive, 160 economy class tickets , maximum profit = Rs. 136000
33. 2ab sq.units 34. $\frac{1}{4}(\pi - 2)$ sq.units, $\frac{4}{3}(8\pi - \sqrt{3})$ sq. units
35. $x = \tan^{-1}y - 1 + Ce^{-\tan^{-1}y}$, $\cot \frac{y}{x} = \log|x| + C$
36. $\frac{x-2}{6} = \frac{y-4}{3} = \frac{z+1}{2}$, (-4, 1, -3)

**SET-1****Series BVM/2**कोड नं. **65/2/1**
Code No.रोल नं.

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Roll No.

परीक्षार्थी कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Code on the title page of the answer-book.

- कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **11** हैं ।
- प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए कोड नम्बर को छात्र उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।
- कृपया जाँच कर लें कि इस प्रश्न-पत्र में **29** प्रश्न हैं ।
- कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, प्रश्न का क्रमांक अवश्य लिखें ।
- इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।
- Please check that this question paper contains **11** printed pages.
- Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains **29** questions.
- **Please write down the Serial Number of the question before attempting it.**
- 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

गणित**MATHEMATICS**

निर्धारित समय : 3 घण्टे

अधिकतम अंक : 100

Time allowed : 3 hours

Maximum Marks : 100

65/2/1

1

P.T.O.



सामान्य निर्देश:

- सभी प्रश्न अनिवार्य हैं।
- इस प्रश्न-पत्र में 29 प्रश्न हैं जो चार खण्डों में विभाजित हैं: अ, ब, स तथा द। खण्ड अ में 4 प्रश्न हैं जिनमें से प्रत्येक एक अंक का है। खण्ड ब में 8 प्रश्न हैं जिनमें से प्रत्येक दो अंक का है। खण्ड स में 11 प्रश्न हैं जिनमें से प्रत्येक चार अंक का है। खण्ड द में 6 प्रश्न हैं जिनमें से प्रत्येक छः अंक का है।
- खण्ड अ में सभी प्रश्नों के उत्तर एक शब्द, एक वाक्य अथवा प्रश्न की आवश्यकतानुसार दिए जा सकते हैं।
- पूर्ण प्रश्न-पत्र में विकल्प नहीं हैं। फिर भी खण्ड अ के 1 प्रश्न में, खण्ड ब के 3 प्रश्नों में, खण्ड स के 3 प्रश्नों में तथा खण्ड द के 3 प्रश्नों में आन्तरिक विकल्प है। ऐसे सभी प्रश्नों में से आपको एक ही विकल्प हल करना है।
- कैलकुलेटर के प्रयोग की अनुमति नहीं है। यदि आवश्यक हो, तो आप लघुगणकीय सारणियाँ माँग सकते हैं।

General Instructions :

- All questions are compulsory.
- The question paper consists of 29 questions divided into four sections A, B, C and D. Section A comprises of 4 questions of one mark each, Section B comprises of 8 questions of two marks each, Section C comprises of 11 questions of four marks each and Section D comprises of 6 questions of six marks each.
- All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
- There is no overall choice. However, internal choice has been provided in 1 question of Section A, 3 questions of Section B, 3 questions of Section C and 3 questions of Section D. You have to attempt only one of the alternatives in all such questions.
- Use of calculators is not permitted. You may ask for logarithmic tables, if required.

खण्ड अ

SECTION A

प्रश्न संख्या 1 से 4 तक प्रत्येक प्रश्न 1 अंक का है।

Question numbers 1 to 4 carry 1 mark each.

- यदि A एक वर्ग आव्यूह है जिसमें $A'A = I$ है, तो $|A|$ का मान लिखिए।
If A is a square matrix satisfying $A'A = I$, write the value of $|A|$.
- यदि $y = x|x|$ है, तो $x < 0$ के लिए, $\frac{dy}{dx}$ ज्ञात कीजिए।
If $y = x|x|$, find $\frac{dy}{dx}$ for $x < 0$.
- निम्न अवकल समीकरण की कोटि व घात (यदि परिभाषित है) ज्ञात कीजिए :

$$\frac{d^2y}{dx^2} + x \left(\frac{dy}{dx} \right)^2 = 2x^2 \log \left(\frac{d^2y}{dx^2} \right)$$

Find the order and degree (if defined) of the differential equation

$$\frac{d^2y}{dx^2} + x \left(\frac{dy}{dx} \right)^2 = 2x^2 \log \left(\frac{d^2y}{dx^2} \right)$$



4. उस रेखा के दिक्-कोसाइन ज्ञात कीजिए जो निर्देशांक अक्षों से समान कोण बनाती है ।

अथवा

एक रेखा किसी एक बिन्दु, जिसका स्थिति सदिश $2\hat{i} - \hat{j} + 4\hat{k}$ है, से गुजरती है और सदिश $\hat{i} + \hat{j} - 2\hat{k}$ की दिशा में है । इस रेखा का कार्तीय समीकरण ज्ञात कीजिए ।

Find the direction cosines of a line which makes equal angles with the coordinate axes.

OR

A line passes through the point with position vector $2\hat{i} - \hat{j} + 4\hat{k}$ and is in the direction of the vector $\hat{i} + \hat{j} - 2\hat{k}$. Find the equation of the line in cartesian form.

खण्ड ब

SECTION B

प्रश्न संख्या 5 से 12 तक प्रत्येक प्रश्न के 2 अंक हैं ।

Question numbers 5 to 12 carry 2 marks each.

5. सभी वास्तविक संख्याओं के समुच्चय \mathbb{R} पर परिभाषित संक्रिया $*$: $a * b = \sqrt{a^2 + b^2}$ क्या द्विआधारी है, इसकी जाँच कीजिए । यदि यह द्विआधारी है, तो ज्ञात कीजिए कि क्या यह साहचर्य है या नहीं ।

Examine whether the operation $*$ defined on \mathbb{R} , the set of all real numbers, by $a * b = \sqrt{a^2 + b^2}$ is a binary operation or not, and if it is a binary operation, find whether it is associative or not.

6. यदि $A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$ है, तो दर्शाइए कि $(A - 2I)(A - 3I) = 0$.

If $A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$, show that $(A - 2I)(A - 3I) = 0$.

7. ज्ञात कीजिए :

$$\int \sqrt{3 - 2x - x^2} dx$$

Find :

$$\int \sqrt{3 - 2x - x^2} dx$$



8. ज्ञात कीजिए :

$$\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} dx$$

अथवा

ज्ञात कीजिए :

$$\int \frac{x-3}{(x-1)^3} e^x dx$$

Find :

$$\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} dx$$

OR

Find :

$$\int \frac{x-3}{(x-1)^3} e^x dx$$

9. वक्रों के कुल $y = Ae^{2x} + Be^{-2x}$, जहाँ A, B स्वेच्छ अचर हैं, को निरूपित करने वाला अवकल समीकरण ज्ञात कीजिए ।

Find the differential equation of the family of curves $y = Ae^{2x} + Be^{-2x}$, where A and B are arbitrary constants.

10. यदि $|\vec{a}| = 2$, $|\vec{b}| = 7$ तथा $\vec{a} \times \vec{b} = 3\hat{i} + 2\hat{j} + 6\hat{k}$ है, तो \vec{a} और \vec{b} के बीच का कोण ज्ञात कीजिए ।

अथवा

उस घनाभ का आयतन ज्ञात कीजिए जिसके किनारे $-3\hat{i} + 7\hat{j} + 5\hat{k}$, $-5\hat{i} + 7\hat{j} - 3\hat{k}$ तथा $7\hat{i} - 5\hat{j} - 3\hat{k}$ द्वारा दिए गए हैं ।

If $|\vec{a}| = 2$, $|\vec{b}| = 7$ and $\vec{a} \times \vec{b} = 3\hat{i} + 2\hat{j} + 6\hat{k}$, find the angle between \vec{a} and \vec{b} .

OR

Find the volume of a cuboid whose edges are given by $-3\hat{i} + 7\hat{j} + 5\hat{k}$, $-5\hat{i} + 7\hat{j} - 3\hat{k}$ and $7\hat{i} - 5\hat{j} - 3\hat{k}$.

11. यदि $P(A \text{ नहीं}) = 0.7$, $P(B) = 0.7$ तथा $P(B/A) = 0.5$ है, तो $P(A/B)$ ज्ञात कीजिए ।
If $P(\text{not } A) = 0.7$, $P(B) = 0.7$ and $P(B/A) = 0.5$, then find $P(A/B)$.



12. एक सिक्का 5 बार उछाला गया । (i) 3 बार चित आने की प्रायिकता क्या है ?
(ii) अधिकतम 3 बार चित आने की प्रायिकता क्या है ?

अथवा

दो सिक्कों को एक बार एक साथ उछालने पर चितों की संख्या, X का प्रायिकता बंटन ज्ञात कीजिए ।

A coin is tossed 5 times. What is the probability of getting (i) 3 heads, (ii) at most 3 heads ?

OR

Find the probability distribution of X, the number of heads in a simultaneous toss of two coins.

खण्ड स

SECTION C

प्रश्न संख्या 13 से 23 तक प्रत्येक प्रश्न के 4 अंक हैं ।

Question numbers 13 to 23 carry 4 marks each.

13. जाँच कीजिए कि क्या समुच्चय $A = \{1, 2, 3, 4, 5, 6\}$ पर परिभाषित संबंध $R = \{(a, b) : b = a + 1\}$ स्वतुल्य, सममित या संक्रामक है ।

अथवा

मान लीजिए कि $f : N \rightarrow Y$, $f(x) = 4x + 3$, द्वारा परिभाषित एक फलन है, जहाँ $Y = \{y \in N : y = 4x + 3, \text{ किसी } x \in N \text{ के लिए}\}$ है । सिद्ध कीजिए कि f व्युत्क्रमणीय है । इसका प्रतिलोम फलन भी ज्ञात कीजिए ।

Check whether the relation R defined on the set $A = \{1, 2, 3, 4, 5, 6\}$ as $R = \{(a, b) : b = a + 1\}$ is reflexive, symmetric or transitive.

OR

Let $f : N \rightarrow Y$ be a function defined as $f(x) = 4x + 3$,

where $Y = \{y \in N : y = 4x + 3, \text{ for some } x \in N\}$. Show that f is invertible.

Find its inverse.

14. $\sin\left(\cos^{-1}\frac{4}{5} + \tan^{-1}\frac{2}{3}\right)$ का मान ज्ञात कीजिए ।

Find the value of $\sin\left(\cos^{-1}\frac{4}{5} + \tan^{-1}\frac{2}{3}\right)$.

15. सारणिकों के गुणधर्मों का प्रयोग करके, दर्शाइए कि

$$\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix} = 3(a+b+c)(ab+bc+ca)$$



Using properties of determinants, show that

$$\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix} = 3(a+b+c)(ab+bc+ca)$$

16. यदि $x\sqrt{1+y} + y\sqrt{1+x} = 0$ और $x \neq y$ है, तो सिद्ध कीजिए कि $\frac{dy}{dx} = -\frac{1}{(x+1)^2}$.

अथवा

यदि $(\cos x)^y = (\sin y)^x$ है, तो $\frac{dy}{dx}$ ज्ञात कीजिए।

If $x\sqrt{1+y} + y\sqrt{1+x} = 0$ and $x \neq y$, prove that $\frac{dy}{dx} = -\frac{1}{(x+1)^2}$.

OR

If $(\cos x)^y = (\sin y)^x$, find $\frac{dy}{dx}$.

17. यदि, किसी $c > 0$ के लिए, $(x-a)^2 + (y-b)^2 = c^2$ है, तो सिद्ध कीजिए कि

$$\frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}, \text{ a और b से स्वतंत्र एक स्थिर राशि है।}$$

If $(x-a)^2 + (y-b)^2 = c^2$, for some $c > 0$, prove that

$$\frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} \text{ is a constant independent of a and b.}$$

18. वक्र $x^2 = 4y$ पर उस अभिलंब का समीकरण ज्ञात कीजिए, जो बिन्दु $(-1, 4)$ से गुजरता है।

Find the equation of the normal to the curve $x^2 = 4y$ which passes through the point $(-1, 4)$.



19. ज्ञात कीजिए :

$$\int \frac{x^2 + x + 1}{(x + 2)(x^2 + 1)} dx$$

Find :

$$\int \frac{x^2 + x + 1}{(x + 2)(x^2 + 1)} dx$$

20. सिद्ध कीजिए कि

$$\int_0^a f(x) dx = \int_0^a f(a - x) dx$$

अतः

$$\int_0^{\pi/2} \frac{x}{\sin x + \cos x} dx$$

का मूल्यांकन कीजिए ।

Prove that

$$\int_0^a f(x) dx = \int_0^a f(a - x) dx$$

and hence evaluate

$$\int_0^{\pi/2} \frac{x}{\sin x + \cos x} dx$$

21. अवकल समीकरण को हल कीजिए :

$$x \frac{dy}{dx} = y - x \tan \left(\frac{y}{x} \right)$$

अथवा

अवकल समीकरण को हल कीजिए :

$$\frac{dy}{dx} = - \left[\frac{x + y \cos x}{1 + \sin x} \right]$$



Solve the differential equation :

$$x \frac{dy}{dx} = y - x \tan \left(\frac{y}{x} \right)$$

OR

Solve the differential equation :

$$\frac{dy}{dx} = - \left[\frac{x + y \cos x}{1 + \sin x} \right]$$

22. सदिशों $\vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ और $\vec{c} = \lambda\hat{i} + 2\hat{j} + 3\hat{k}$ के लिए, सदिश $\vec{b} + \vec{c}$ के अनुदिश मात्रक सदिश व सदिश $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ का अदिश गुणनफल 1 है। λ का मान ज्ञात कीजिए और अतः $\vec{b} + \vec{c}$ के अनुदिश मात्रक सदिश भी ज्ञात कीजिए।

The scalar product of the vector $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of the vectors $\vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\vec{c} = \lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to 1. Find the value of λ and hence find the unit vector along $\vec{b} + \vec{c}$.

23. यदि रेखाएँ $\frac{x-1}{-3} = \frac{y-2}{2\lambda} = \frac{z-3}{2}$ और $\frac{x-1}{3\lambda} = \frac{y-1}{2} = \frac{z-6}{-5}$ परस्पर लम्बवत् हों, तो λ का मान ज्ञात कीजिए। अतः ज्ञात कीजिए कि क्या ये रेखाएँ एक-दूसरे को काटती हैं या नहीं।

If the lines $\frac{x-1}{-3} = \frac{y-2}{2\lambda} = \frac{z-3}{2}$ and $\frac{x-1}{3\lambda} = \frac{y-1}{2} = \frac{z-6}{-5}$ are perpendicular, find the value of λ . Hence find whether the lines are intersecting or not.



खण्ड द
SECTION D

प्रश्न संख्या 24 से 29 तक प्रत्येक प्रश्न के 6 अंक हैं।

Question numbers 24 to 29 carry 6 marks each.

24. यदि $A = \begin{bmatrix} 1 & 3 & 4 \\ 2 & 1 & 2 \\ 5 & 1 & 1 \end{bmatrix}$ है, तो A^{-1} ज्ञात कीजिए।

अतः निम्न समीकरण निकाय का हल ज्ञात कीजिए :

$$x + 3y + 4z = 8$$

$$2x + y + 2z = 5$$

और $5x + y + z = 7$

अथवा

प्रारंभिक रूपांतरणों द्वारा, निम्न आव्यूह का व्युत्क्रम ज्ञात कीजिए :

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

If $A = \begin{bmatrix} 1 & 3 & 4 \\ 2 & 1 & 2 \\ 5 & 1 & 1 \end{bmatrix}$, find A^{-1} .

Hence solve the system of equations

$$x + 3y + 4z = 8$$

$$2x + y + 2z = 5$$

and $5x + y + z = 7$

OR



Find the inverse of the following matrix, using elementary transformations :

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

25. सिद्ध कीजिए कि एक R त्रिज्या के गोले के अंतर्गत अधिकतम आयतन के बेलन की ऊँचाई $\frac{2R}{\sqrt{3}}$ है। अधिकतम आयतन भी ज्ञात कीजिए।

Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find the maximum volume.

26. समाकलन विधि से उस त्रिभुज का क्षेत्रफल ज्ञात कीजिए जिसके शीर्ष (1, 0), (2, 2) और (3, 1) हैं।

अथवा

समाकलन विधि से, दो वृत्तों $x^2 + y^2 = 4$ तथा $(x - 2)^2 + y^2 = 4$ के बीच घिरे क्षेत्र का क्षेत्रफल ज्ञात कीजिए।

Using method of integration, find the area of the triangle whose vertices are (1, 0), (2, 2) and (3, 1).

OR

Using method of integration, find the area of the region enclosed between two circles $x^2 + y^2 = 4$ and $(x - 2)^2 + y^2 = 4$.

27. बिन्दुओं, जिनके स्थिति सदिश $\hat{i} + \hat{j} - 2\hat{k}$, $2\hat{i} - \hat{j} + \hat{k}$ तथा $\hat{i} + 2\hat{j} + \hat{k}$ हैं, से गुज़रने वाले समतल का सदिश व कार्तीय समीकरण ज्ञात कीजिए। उपर्युक्त समतल के समांतर समतल, जो बिन्दु (2, 3, 7) से गुज़रता है, का समीकरण भी लिखिए। अतः, दोनों समांतर समतलों के बीच की दूरी ज्ञात कीजिए।

अथवा

बिन्दुओं (2, -1, 2) तथा (5, 3, 4) से गुज़रने वाली रेखा का समीकरण ज्ञात कीजिए तथा बिन्दुओं (2, 0, 3), (1, 1, 5) तथा (3, 2, 4) से गुज़रने वाले समतल का समीकरण भी ज्ञात कीजिए। रेखा व समतल का प्रतिच्छेदन बिन्दु भी ज्ञात कीजिए।



Find the vector and cartesian equations of the plane passing through the points having position vectors $\hat{i} + \hat{j} - 2\hat{k}$, $2\hat{i} - \hat{j} + \hat{k}$ and $\hat{i} + 2\hat{j} + \hat{k}$. Write the equation of a plane passing through a point (2, 3, 7) and parallel to the plane obtained above. Hence, find the distance between the two parallel planes.

OR

Find the equation of the line passing through (2, -1, 2) and (5, 3, 4) and of the plane passing through (2, 0, 3), (1, 1, 5) and (3, 2, 4). Also, find their point of intersection.

28. तीन सिक्के दिए गए हैं। एक सिक्के के दोनों ओर चित ही है। दूसरा सिक्का अभिनत है जिसमें चित 75% बार प्रकट होता है और तीसरा अनभिनत सिक्का है। तीनों में से एक सिक्का यादृच्छया चुना गया और उसे उछाला गया है। यदि सिक्के पर चित प्रकट हुआ हो, तो क्या प्रायिकता है कि वह दोनों तरफ चित वाला सिक्का है ?

There are three coins. One is a two-headed coin, another is a biased coin that comes up heads 75% of the time and the third is an unbiased coin. One of the three coins is chosen at random and tossed. If it shows heads, what is the probability that it is the two-headed coin ?

29. एक कंपनी दो प्रकार का सामान, A और B बनाती है, जिसमें सोने व चाँदी का उपयोग होता है। प्रकार A की प्रत्येक इकाई में 3 g चाँदी व 1 g सोना, तथा प्रकार B की प्रत्येक इकाई में 1 g चाँदी व 2 g सोना प्रयोग में आता है। कंपनी ज्यादा-से-ज्यादा 9 g चाँदी व 8 g सोने का ही प्रयोग कर सकती है। यदि प्रकार A की एक इकाई से ₹ 40 का लाभ व प्रकार B की एक इकाई से ₹ 50 का लाभ कमाया जाता है, तो अधिकतम लाभ अर्जित करने हेतु कंपनी को दोनों प्रकारों की कितनी-कितनी इकाइयाँ बनानी चाहिएँ ? उपर्युक्त समस्या को रैखिक प्रोग्रामन समस्या में परिवर्तित करके आलेख विधि से हल कीजिए तथा अधिकतम लाभ भी ज्ञात कीजिए।

A company produces two types of goods, A and B, that require gold and silver. Each unit of type A requires 3 g of silver and 1 g of gold while that of type B requires 1 g of silver and 2 g of gold. The company can use at the most 9 g of silver and 8 g of gold. If each unit of type A brings a profit of ₹ 40 and that of type B ₹ 50, find the number of units of each type that the company should produce to maximize profit. Formulate the above LPP and solve it graphically and also find the maximum profit.



तत् त्वं पूषन् अपावृणु
केन्द्रीय विद्यालय संगठन

केन्द्रीय विद्यालय संगठन

KENDRIYA VIDYALAYA SANGATHAN

18, संस्थागत क्षेत्र, शहीद जीत सिंह मार्ग, नई दिल्ली-110016

18, Institutional Area, Shaheed Jeet Singh Marg, New Delhi-110016

Website : www.kvsangathan.nic.in

 @KVSHQ  @KVS_HQ